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THE PHOTO-ELECTRIC AND THERMIONIC PROPERTIES OF IRON

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The photo-electric properties of metals which have undergone a thorough outgassing process by heat treatment have not been extensively studied. DuBridge,[1] Warner[2] and Kazda[3] have studied the photo-electric characteristics of platinum, tungsten and mercury, respectively, taking particular outgassing precautions. An effort is being made in this laboratory to extend the study to other metals. The investigations reported in the present paper have yielded some interesting results for pure iron. Others in this laboratory are at the present time working with molybdenum, tantalum and cobalt.

The present paper is a preliminary report of investigations of the following phenomena:

(1) The variation of the photo-electric sensitivity of iron as it undergoes an extended process of outgassing by heat treatment.

(2) The effect of crystallographic changes on the photo-electric sensitivity of thoroughly outgassed iron.

(3) The effect of a crystallographic change on the thermionic emission from outgassed iron.

(4) The variation of the long wave-length limit through the outgassing process.

The apparatus was similar to that used by DuBridge[1] and consisted essentially of a narrow strip of iron suspended inside a nickel receiving cylinder, the whole being enclosed in a Pyrex tube which was connected through a liquid air trap and a mercury cut-off to the pumps. A quartz window, sealed directly on the pyrex tube served to admit the radiations from a quartz mercury arc. The pressures could be measured by a McLeod gauge or an ionization manometer of the type described by Dushman and Found.[4]

The electrolytic iron, used in this work, was supplied by Professor Watts of the chemical engineering department of the University of Wis-

consin. After being carefully annealed in a vacuum (10^{-3} mm. of Hg), the iron was rolled into strips 0.02 mm. thick, 3 mm. wide and about 10 cm. long. These strips were in some cases cleaned with dehydrated alcohol before being suspended in the form of a loop in the nickel receiving cylinder, but such cleaning produced no essential change in the characteristics.

The photo-electric currents, produced by the radiation from the arc, were measured by a Compton electrometer having a sensitivity of 300 mm. per volt. A resistance of 10^8 ohms shunted across the quadrants permitted the currents to be measured by the "steady deflection" method.

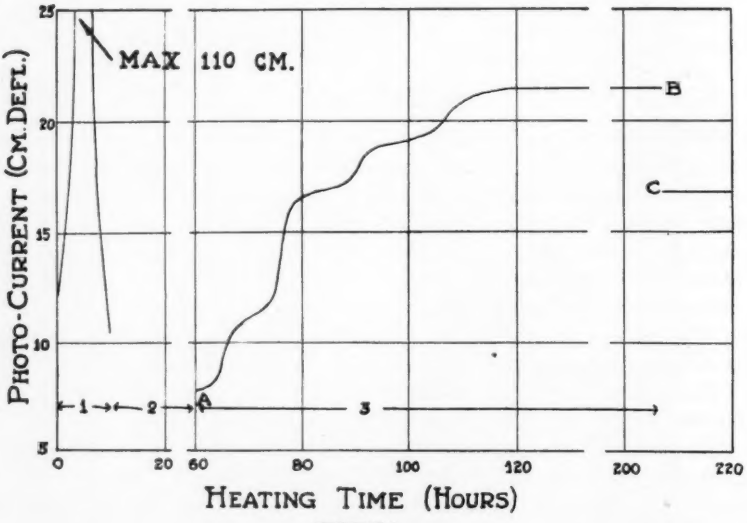

FIGURE 1

Interval 1 (0 to 10 hrs.), heated filament at 300°C. Interval 2 (10 to 60 hrs.), baked entire tube at 550°C. Interval 3 (60 to 206 hrs.), glowed filament at 875°C. Drop in sensitivity at C due to supercooling from above 910°C.

Pressures.—During the initial stages of outgassing the ionization gauge showed great increases of the pressure in the tube when the filament was heated. These increases in pressure became smaller as the outgassing progressed, and finally no change could be observed when the filament was heated. The final pressure obtained was 1×10^{-8} mm. of Hg.

Variation of Photo-Electric Current with Time of Outgassing.—The data for figure 1, which shows the effect of heat treatment on the photo-electric sensitivity for one particular specimen, were taken immediately after the heating currents had been cut off. During interval 1 there was a tremendous increase in the photo-electric sensitivity followed by an equally great decrease. No photo-currents were measured during interval 2

while the entire tube was being baked at 550°C. However, the photo-electric sensitivity was much less after this baking interval. (Point *A*, Fig. 1.) The photo-electric current increased in rather abrupt steps during interval 3, finally reaching a value which did not change with additional heat treatment at the same temperature. A probable reason for these abrupt changes will be pointed out later. As outgassing progressed the photo-electric sensitivity changed less rapidly with time, while standing cold, until finally a thoroughly outgassed specimen showed no fatigue for 12 hours after heat treatment. The time required for outgassing varied a great deal for the different specimens. However, the general behavior was in every case the same.

After this condition of stability had been attained, it was found that the magnitude of the photo-electric current depended on the rate of cooling the filament. An iron surface having a photo-electric sensitivity near the value of *C* (Fig. 1) was produced when the filament was suddenly cooled from above 910°C. to room temperature. To again produce photo-currents near the value of *B* (Fig. 1) it was only necessary to cool the filament more slowly to room temperature or to cool it more slowly to below 910°C. and then suddenly to room temperature. Surfaces having either of these sensitivities could be obtained even after more than 100 hours of additional heat treatment. For reasons which will be pointed out later, it may be said that this decrease in photo-electric sensitivity which occurred when the filament was cooled suddenly from above 910°C. was due to the retention of a part of the γ iron.

Variation of Photo-Electric Sensitivity with Temperature.—The data for figure 2 were obtained while the filament was hot and after it had been thoroughly outgassed by more than 150 hours of heat treatment. The curve shows the variation of the photo-sensitivity *with the temperature at which observations were made.* As it was impossible to use the optical pyrometer for measuring temperatures below 875°C., the photo-current has been plotted as a function of the heating current through the filament. The temperatures corresponding to three heating currents have been indicated in the figure. These temperatures were located by comparing a resistance-heating current curve with the resistance-temperature curve obtained by Burgess and Kellberg.[5] The resistance of the leads from the seals to the filament was so small compared with the resistance of the filament that, in addition to the change in slope at 768°C. and 910°C., it was possible to obtain the small "resistance-hysteresis" effect near the transition from β to γ iron. The optical pyrometer was also used to check the point at 910°C.

Curves 3 and 4 were taken while the pressure was 1×10^{-8} mm. of Hg; curves 1 and 2 were taken after the pressure had been increased to 5×10^{-6} mm. of Hg. The increase in pressure was produced by heating

the glass parts of the tube with a flame after the pumps had been disconnected from the system by a mercury cut-off. The arrows on the curves indicate increasing and decreasing heating currents.

Many such curves have been obtained using two different specimens and, although the absolute magnitude of the change varied slightly, in no case did the curves fail to show the change indicated by the curves in figure 2. Due to the large thermionic emission, measurements of the photo-electric currents at higher temperatures have up to the present time not been successful.

Thermionic Current.—If the thermionic current is plotted as a function of the heating current (temperature) as is done in figure 3, there is, at a heating current corresponding to 910°C., a departure from the ordinary thermionic-temperature curve. The temperature indicated in the figure is the average and therefore due to the end losses, parts of the filament reached the transition point at a slightly lower heating current. Several such curves have been obtained and each showed the change near 910°C.

Long Wave-Length Limit.—The long wave limit for thoroughly outgassed iron was found to be between 2580 Å and 2652 Å. These limits were determined by the use of absorption cells in the path of the incident light. Photographs taken with a quartz spectrograph showed that the mercury line λ 2652 Å was transmitted quite strongly by a solution which absorbed all the photo-electrically effective energy. The energy transmitted by a solution which cut off sharply just below the mercury line λ 2580 Å produced a photo-current of 0.5 mm. The electrometer sensitivity was increased eleven-fold for these observations.

If these limits are substituted in Einstein's photo-electric equation, the work-function for thoroughly outgassed iron is found to be 4.72 ± 0.07 volts.

No change from the above value of the photo-electric threshold could be detected for iron which had been cooled suddenly from above 910°C. In view of the fact (pointed out in connection with the discussion of Fig. 1) that only *a part* of the γ iron was retained in such a filament, it would hardly be expected that the threshold would be different from that of α or β iron, since the latter forms probably have, if anything, a smaller work function than the γ form. Attempts at determining the threshold for iron at temperatures above the transition points have, so far, not been successful.

The long wave limit was measured several times as the outgassing progressed. The limit increased to a value above 3000 Å at the end of five hours' heat treatment (see Fig. 1). Accompanying the decrease in the photo-electric sensitivity which was caused by further heat treatment, a corresponding decrease in the long wave limit was observed, a value as low as 2400 Å having been obtained after the tube had been baked at 550°C. (Point *A*, Fig. 1.)

It has been pointed out above that the threshold increased from 2400 Å to between 2580 and 2652 Å during the heating interval, 60 to 120 hours (Fig. 1). It is therefore believed that the abrupt increases in the photo-electric sensitivity in this interval occur as new lines in the irradiating light become effective.

Welch[6] has recently reported a long wave limit of 3150 Å for iron, the surface of which was filed in a vacuum, but the specimen was not thoroughly heat treated. Apparently Welch's filing process gave him a surface having approximately the same characteristics as are produced by a few hours' heating in a vacuum (see maximum point, Fig. 1, where

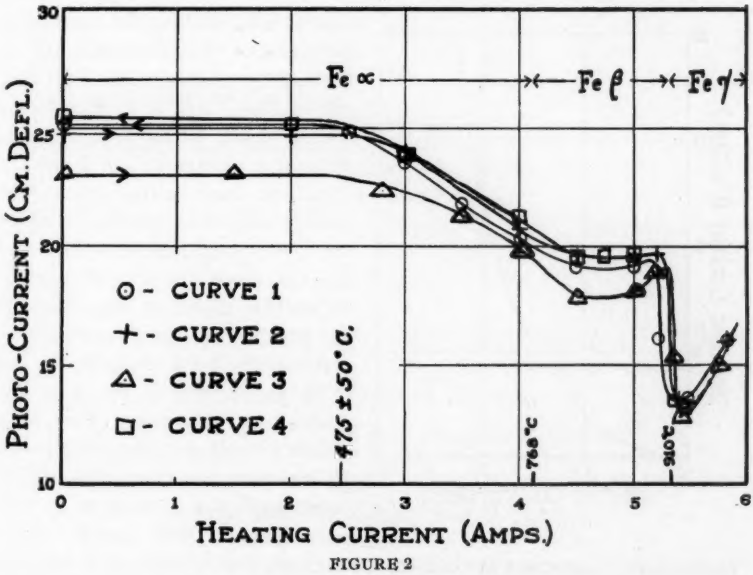

FIGURE 2

threshold was above 3000 Å). Furthermore, Welch reports a photo-electric fatigue, which was in no case observed with thoroughly outgassed specimens during the present work. His results, therefore, cannot be taken as characteristic of gas-free metals.

Discussion of Results.—It is seen from figure 2 that the variation in the photo-electric current with temperature is quite complex. Between about 475°C. and 768°C. there is a decrease in the sensitivity which is perhaps a pure temperature effect analogous to the *increase* obtained for platinum by DuBridge.[1] At 768°C. a new factor enters and the sensitivity either remains constant or increases slightly up to 910°C. This change is obviously closely associated with the transition, at 768°C., from α to β iron,

which involves a slight increase in the edge of the unit cube but no change in crystal form. Near 910°C., where the crystal structure changes from the body centered cubic to the face centered cubic type, there is an abrupt decrease in the photo-electric sensitivity. Beyond this transition from β to γ iron, there is an increase in the sensitivity. A radical change at the β to γ transition is consistent with the observation of Burgess and Scott[7] upon the thermoelectric power of iron. They found that at the α to β transition point there is only a change in the slope of the thermo-electric power curve, while there is a marked drop in the curve at the β to γ transition.

The thermionic curve (Fig. 3) is an interesting extension of the work of Goetz[8] who studied the thermionic properties of iron from 1100°C. through the melting point. He concludes that there is a decrease in the N of the Richardson equation without a change in the Φ, at the transition from γ to δ iron. The present apparatus was not designed for thermionic measurements and this, combined with temperature uncertainties, makes it impossible to say whether the change near 910°C. corresponds to a *decrease* in the N or an *increase* in the Φ of the Richardson equation. The work will be extended to clear this point. If, however, the change at 910°C. corresponds to a decrease in N, then a comparison with Goetz's work shows that N decreases at both the transitions, β to γ and γ to δ, whereas the crystallographic changes are exactly opposite in the two cases.

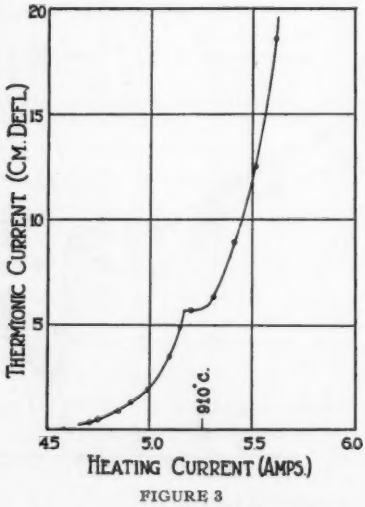

FIGURE 3

While there can be no doubt that the changes in the photo-electric sensitivity with temperature and the break in the thermionic curve are associated with the known transitions in the iron, it is possible to take the point of view that these are indirect rather than direct results of such transitions. It is possible to take the position that the specimen was at no time free from gas and to assume that there is a characteristic equilibrium value of the gas content of the surface which is a function of the temperature and also of the crystal structure resulting in the observed variations of the photo-electric sensitivity. The curves of figure 2, which were taken at greatly increased pressures, are evidence against this point of view because they show that the above hypothetical equilibrium gas

layer is not altered by a five-hundred fold change in the surrounding pressure. Further evidence for believing that a condition characteristic of the metal itself has been reached is furnished by the fact that the photoelectric sensitivity of an outgassed specimen remains constant for twelve hours after heat treatment. The work of Davisson and Germer[9] on electron reflection from nickel is also against such a point of view. They found that it takes several hours for the characteristic reflection to be obscured by the accumulation of gas on the surface.

The work is being continued and a more complete report will be published soon.

In conclusion, the author desires to acknowledge his appreciation to Dr. C. E. Mendenhall, under whose direction the work was carried on.

[1] DuBridge, *Phys. Rev.*, **29**, 451 (1927).
[2] Warner, *Proc. Nat. Acad. Sci.*, **13**, 56 (1927).
[3] Kazda, *Phys. Rev.*, **26**, 643 (1925).
[4] Dushman and Found, *Phys. Rev.*, **23**, 734 (1924).
[5] Burgess and Kellberg, *Bull. Bur. Stds.*, **11**, 457 (1914–15).
[6] Welch, Abstr. 19, N. Y. Meeting Amer. Phys. Soc. (1928).
[7] Burgess and Scott, *Bull. Bur. Stds.*, **14**, 15 (1918).
[8] Goetz, *Phys. Zeit.*, **24**, 379 (1924).
[9] Davisson and Germer, *Phys. Rev.*, **30**, 705 (1927).

EVIDENCE FOR THE CONTINUOUS CREATION OF THE COMMON ELEMENTS OUT OF POSITIVE AND NEGATIVE ELECTRONS

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Read before the Academy April 23, 1928

The evidence obtained from the study of cosmic rays that the more stable and more abundant elements like helium (abundant in the heavens), oxygen, silicon and iron, are being formed *at the present time* out of the primordial positive and negative electrons, the former of which is the nucleus of the hydrogen atom, may be briefly summarized as follows:

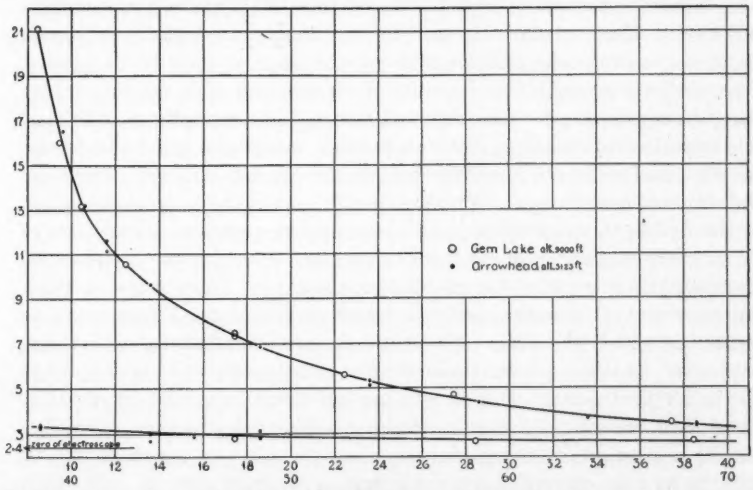
First.—The pilot-balloon experiments of Millikan and Bowen,[1] in which they sent up recording electroscopes 0.92 of the way to the top of the atmosphere and in which the absorption coefficient of the cosmic rays at, or near, the top of the atmosphere came out of the same order of magnitude as that found near sea-level,[2] show conclusively that these rays consist of a definite and distinct region of spectral frequencies, or oscillations, a hundred times more rapid than those produced by the most powerful subatomic changes heretofore known, namely, those accompanying radio-

active processes. Otherwise stated, these experiments show conclusively that there are no radiations of appreciable intensity entering the earth's atmosphere having frequencies intermediate between those of the gamma rays and those of the cosmic rays. For, since the hardest gamma rays are capable of penetrating a thickness of about 70 cm. of water, while, as indicated below, the cosmic rays are capable of penetrating 70 m. of water, and since penetrating power increases approximately as frequency, if rays of appreciable intensity came into the atmosphere having frequencies between those of gamma rays and those of the cosmic rays they would of necessity have caused the rapid discharge of an electroscope which rose to within the equivalent of 80 cm. of water of the top of the atmosphere, the whole of the earth's atmosphere being the equivalent of 10 m. of water. No such rapid discharge took place; hence there are no *strong* radiations entering the earth's atmosphere in that particular region of frequencies.

Second.—The experiments of Millikan and Cameron of the last Summer and Fall made in deep, high-altitude California lakes, with new electroscopes eight times more sensitive than those the authors had theretofore used, brought to light the definite proof that the cosmic-ray spectrum consists of definite bands, like those of neon or mercury lamps, containing spectral lines as much as three octaves apart, the highest frequency band having so enormous a penetrating power that it passes through as much as 200 feet of water (18 feet of lead) before becoming completely absorbed. This discovery of a banded structure in the cosmic rays shows that they are not produced as are x-rays by the impact upon the atoms of matter of electrons which have acquired large velocities by falling through powerful electrical fields as we earlier suggested[3]—the fields needed to produce frequencies of the order of the cosmic rays would be at least 150,000,000 volts—(3000 times as great as the field existing in x-ray tubes)—but that *they are rather produced by definite and continually recurring atomic transformations involving very much greater energy-changes than any occurring in radio-active processes.* The proof of the banded structure of the cosmic-ray spectrum is found in the fact that the new ionization-depth curve, (figure 1) which is seen to be of very much higher resolving power than any heretofore obtained, when taken in connection with the sounding balloon data, shows a nearly constant absorption coefficient from close to the top of the atmosphere down to about sea-level, 10 m. below the top, then bends quite suddenly and uncovers below 15 m. a new absorption coefficient, of only about one-sixth the former value, which continues down to 70 m. below the top with but little further change. The cosmic rays, therefore, consist of at least two, possibly more (see below), radiation bands of absorption coefficients (and therefore, roughly, also of frequencies) in the ratio 6 to 1. The sharpness of the bend in the ionization-depth curve is completely incompatible with any general dis-

tribution of the frequencies of cosmic rays, like that found in white light, or in the general x-radiation which is produced by the bombardment of the atoms of a target by high-speed electrons (cathode rays).

Third.—If the Einstein special theory of relativity may be taken as a sound basis of reasoning—and no results predicted by it have ever thus far been shown to be incorrect, while it has many striking successes to its credit—then it follows that radiant energy can never escape from an atomic system without the disappearance of an equivalent amount of mass from that system, these relations being contained in the now well-known and universally used equation of Einstein (1905) $Mc^2 = E$, where M is mass in grams, c is the velocity of light in centimeters per second and E is



Ionization in cc./sec. against depth in equivalent m. of water beneath top of atmosphere; upper curve 8 m. to 40 m., lower 38 m. to 70 m.

energy in ergs. Now through the recent, very exact work of Aston[4] we know the mass of every one of the atoms with a great deal of certainty, and we can therefore compute the amount of ether-wave energy that can be generated by any sort of atomic transformation that can take place, and knowing this energy we can compute with the aid of the Einstein equation the frequency, and with the aid of the Dirac formula[5] the penetrating powers of the rays resulting from all possible atomic transformations. *Such studies reveal the fact that there are no possible transformations capable of yielding rays of the enormous penetrating power observed by Millikan and Cameron except those corresponding to the building up or creation of the abundant elements like helium, oxygen, silicon and iron out of hydrogen, or in the case of the last two elements out of helium.* The entire annihilation

of hydrogen by the falling completely together of its positive and negative electrons might be an additional possibility, but it can be eliminated in this case for two excellent reasons. The first of these reasons is that there is practically no place whatever for such a radiation to occupy in the observed ionization-depth curve (see Fig. 1), for it would be between four and five times more penetrating than the radiation that has the smallest absorption coefficient mentioned above. The ionization due to it, if it exists, would then have to be included in the 2.4 ions which represents the "zero of the electroscope" as shown in the figure. But this 2.4 ions is only about one-tenth of the observed ionization at the top of the curve, viz., 21 ions, this topmost reading corresponding to a depth of 1 m. below the surface of Gem Lake. So that this hypothetical radiation can have nothing to do with the observed ionization-depth curve much above the reading 2.4, and below it there is of course room only for a radiation relatively negligible in intensity in comparison with the softer rays that are responsible for the observed curve. The second reason is that this hypothetical radiation, if it were present, would of necessity be homogeneous, and could not therefore exhibit the banded structure shown by the observed cosmic rays. Whether then this act of the entire annihilation of the hydrogen atom through the coming into complete coincidence of the positive and negative electrons takes place or not, it can certainly be eliminated as a cause of the *observed* cosmic rays. There remains, then, no other atomic transformation in which sufficient mass disappears to create the observed cosmic rays except the aforementioned *atom-building* processes. It is important to note that no step-by-step process of building up, or for that matter of disintegrating of atoms, in which one positive electron or one alpha particle is added or subtracted at a time will suffice for the generation of the cosmic rays, since the Einstein equation tells us that in no case can such a transformation produce rays of more than from one-fourth to one-twenty-fifth of the observed penetrating power. The observed extraordinarily penetrating cosmic rays present, then, when taken in connection with Einstein's equation and Aston's findings not only the first direct evidence that the more abundant elements *are now in process* of being created out of positive and negative electrons, but they also present the first indications as to the general character of the specific act or acts by which the atom-building process goes on. So far we have used only the general or qualitative evidence but it will be seen that from it alone the conclusion is scarcely escapable that the powerful cosmic rays here studied can be produced only by the creation *in a single act*, rather than by a step-by-step process, of some, at least, of the common elements out of the primordial positive and negative electrons.

Fourth.—The evidence herewith obtained is, however, not merely qualitative but fairly accurately *quantitative*. For we analyzed very

carefully our cosmic-ray curve empirically before we called on any theoretical considerations whatever to explain it, and we reported in scientific papers that our observed curve demanded three cosmic-ray bands of absorption coefficients 0.35, 0.08 and 0.04 per meter of water, respectively. It was after this work had been done, reported in seminars, written up and prepared for publication in essentially the form in which it will appear[6] that we set about computing from the foregoing considerations what the theoretical absorption coefficients would be if our observed cosmic rays were produced (1) by the formation in one single act of helium out of hydrogen, (2) by the similar formation of oxygen out of hydrogen and (3) by the formation of silicon out of hydrogen. The results of this computation came out 0.30, 0.075 and 0.043, well within the limits of the resolving power of our curve of the observed values. Further, there are only a few elements so abundant that their formation needs to be considered as the possible source of the observed cosmic rays. For the spectroscopy of the heavens shows a very great abundance everywhere of the gases hydrogen, helium and nebulium. But Bowen[7] has just identified the last as nitrogen and oxygen so that these gaseous elements, hydrogen, helium, nitrogen and oxygen seem to be extraordinarily widely spread through space. As to the elements found in solids the meteorites have 96 per cent of their mass in the four elements, oxygen, magnesium, silicon and iron. Magnesium and silicon are close together in atomic weight, 24 and 28, respectively, so that their formation would constitute but one band, the mean energy of which lies approximately at $\mu = 0.04$. It was this joint band that we above called for convenience the silicon band. Similarly the atomic weights of nitrogen and oxygen are respectively 14 and 16, and the mean absorption coefficient 0.08 corresponds to this joint band. The helium band, most significant of all, corresponds to $\mu = 0.30$. There is no other abundant element except iron, and the formation of this out of hydrogen gives a cosmic ray for which $\mu = 0.021$. The existence of such a radiation helps rather than interferes with the fit of our theoretical and experimental curves, but on account of the lack of resolving power in the lower end of our curve it furnishes no trustworthy evidence that this particular act is the one by which iron is most commonly formed. This uncertainty does not exist however with respect to the bands corresponding to $\mu = 0.30$, $\mu = 0.08$ and $\mu = 0.04$. *This whole work constitutes, then, very powerful evidence that the sort of creative, or atom-building processes discussed above, are continually going on all about us, possibly also even on the earth, and that each such event is broadcast through the heavens in the form of the appropriate cosmic ray.*

[1] Millikan and Bowen, *Phys. Rev.*, **22**, 198 (1923); **27**, 353 (1926).

[2] Millikan and Cameron, *Ibid.*, **31**, 163 (1928).

[3] Millikan and Cameron, *Nature*, **121**, 24 (1928).

[4] Aston, *Proc. Roy. Soc.*, **115**, 487 (1927).
[5] Dirac, *Ibid.*, **109**, 206 (1925).
[6] Millikan and Cameron, *Phys. Rev.*, June, 1928.
[7] Bowen, *Astrophys. Jour.*, **57**, 1 (1928).

THE GENERAL X-RADIATION FROM MERCURY VAPOR

By William Duane

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Communicated May 1, 1928

A recently published note[1] described experiments on the general or continuous spectrum radiation coming from the impacts of electrons against mercury vapor atoms. The electrons fell through a constant difference of potential produced by the high tension storage battery and, therefore, had substantially the same velocities when they struck the atoms. The radiation projected from the impacts at right angles to the stream of electrons did not appear to be homogeneous. After it had come through the thin glass window of the apparatus, 57.5% of it passed through a sheet of aluminum about $^1/_{10}$ of a millimeter thick. If the radiation had had the wave-length of the short wave-length limit of the spectrum corresponding to the voltage on the tube, 62% would have passed through the sheet of aluminum.

The object of the researches described in this note has been to investigate the radiation proceeding in the direction of motion of the electron stream, as well as at right angles to it, and, also, to compare the penetration of both rays with the penetration of the radiation that would come from the impacts of the electrons according to certain theories.

Figure 1 represents a horizontal cross-section of the apparatus employed. The electrons from a hot wire cathode, *C*, pass through an opening into the interior of an anode, consisting of two brass tubes, *A* and *B*, joined together by four curved wires, represented by dotted lines in the figures. The high tension storage battery produced a difference of potential of 11,780 volts, which was determined by measuring the current passing through a manganine wire resistance of 3,019,000 ohms in parallel with the tube. The stream of mercury vapor came up through the vertical tube, *D*, from an electrically heated furnace. It then passed across and down through the vertical tube, *E*. The apparatus was exhausted through the vertical tube, *F*, which communicated with a mercury diffusion pump, not shown in the figure. Although all parts of the apparatus contained at least a small amount of mercury vapor, most of the impacts of electrons against the mercury atoms occurred in the region,

E. Radiation coming from these impacts at right angles to the line of flight of the electrons could be examined after it came through the window, *G*. The radiation coming from the impacts in the direction of motion of the electrons could be examined through the window, *H*. In order to prevent the electrons, themselves, from hitting the window, a strong magnetic field, produced by an electro-magnet not shown in the figure, was applied in the region, *B*. Each beam of rays was examined, both by its photographic and by its ionizing effects. To obtain pin-hole camera photographs of the source of radiation, a sheet of lead with a small hole in it, *K*, was mounted in front of each window in turn. In order properly to direct the radiation coming from the mercury vapor atoms only into the ionization chamber, *I*, thumb screws were attached to the lead sheet by which the pin-hole, *K*, could be moved accurately

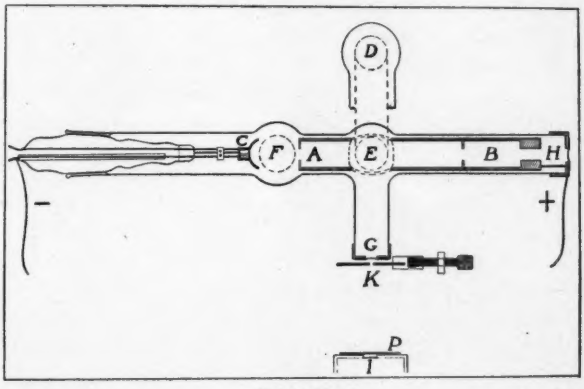
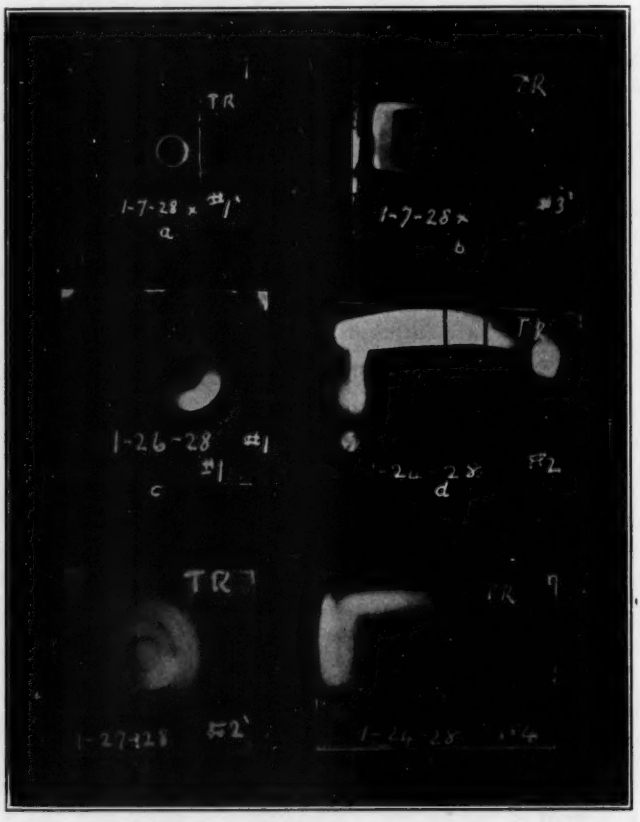


FIGURE 1

in the horizontal and also in the vertical direction. The photographic film, *P*, wrapped in black paper, was placed over the window into the ionization chamber, *I*.

Figure 2 contains copies of several pin-hole camera photographs of the radiation coming through the window, *G;* and also that coming through the window, *H*, after the tube had been turned around so that the window, *H*, lay in front of the pin-hole. Photographs, *a* and *b*, were taken with a very small pin-hole and without the mercury vapor passing through the tube. Photograph *a* represents the source of the radiation passing through the window, *H*. The two circles correspond to the edges of the two circular holes, *A* and *B* (Figure 1), in the anode. The radiation from these edges was produced by a few stray electrons traveling outside of the main beam of electrons, which passed almost wholly through the opening, *A*. Only at one point did the main beam of electrons hit the

edge of A as indicated by the dark spot on one side of the inner circle. Photograph b represents the source of the radiation passing through the window, G. The curved horizontal lines correspond to the wires which connected the portion A to the portion B of the anode. The vertical lines correspond to the edges of the tube, A, and of the tube, B. The radiation producing these photographic impressions came from a few



stray electrons (reflected or secondary) that struck the metal parts of the anode. Photographs, c and d, were taken with a very much larger pin-hole (about 2 mm. in diameter). In these experiments, also, no mercury vapor passed through the tube. Photograph d illustrates the method by which the radiation entering the ionization chamber could be limited to that coming from a particular part, only, of the source of rays. The black lines across the photograph are the shadows of wires

placed directly in front of the photograph, P, on the side toward the tube. The window into the ionization chamber lay directly behind the center of the square formed by the wires. By shifting the position of the pin-hole, K, the radiation from any part of the source of rays could be directed into the ionization chamber. It is important to note that photographs a and c do not show an appreciable effect of radiation inside of the smaller circles. These portions of the photographs correspond to the direct line of flight of the electrons through the tube. The only solid material on this line, inside the tube, was the cathode itself.

Photographs f and g represent the sources of the radiation coming through the windows, H and G, respectively, when the stream of mercury vapor passed through the tube as well as the stream of electrons. Each photograph shows radiation coming from the impacts of electrons against the mercury in addition to that coming from the solid parts of the anode. In photograph f the effect of the mercury radiation lies inside of the inner circle corresponding to the edge of the circular opening, A. The mercury radiation effect appears to be separated from that of the edge by a circular line of less intensity. This line is probably due to the fact that the cathode (a fine focus Coolidge cathode) produced a conical beam of converging electrons and, therefore, its greatest effect lay near the axis of the cone. The radiation producing the outer circle in photograph f came from the edges of the opening, B, in figure 1. The circle is much more strongly marked than in photograph c. It can be seen on the film of c itself, but may not appear in the reproduction. Photograph g represents the effect of the radiation coming at right angles to the stream of electrons through the window, G. In addition to the radiation from the rods and cylinders of the anodes, that from the mercury vapor appears in the center as a sort of elongated cloud. The electron stream passed from right to left and produced its most intense mercury radiation near the center, where the electrons struck the main stream of mercury atoms flowing downward. The cross wires in front of the film cast shadows across this cloud and, as the opening into the ionization chamber lay behind the center of the square, only the radiation from the mercury vapor entered the ionization chamber when the pin-hole, K, was in the position it occupied during the taking of the photograph.

The intensity and penetrating power of the mercury radiation projected in the direction of motion of the electrons and also perpendicular to it have been measured by means of the ionizing effects produced inside of the ionization chamber. These experiments have been made in order to determine whether or not the beams of rays are homogeneous, and, also, to compare the penetration with that to be expected according to certain theoretical distributions of energy in the spectrum of radiation due to the impacts of electrons of given velocity. The material used to de-

termine the penetration of the rays was a sheet of aluminum about 0.1 mm. This test sheet weighed 0.02773 gram per square centimeter. In order to obtain as accurate a comparison between the observed and theoretical penetrations as possible, one must use very thin windows. The windows, G and H, in the tube and the window, I, in the ionization chamber were of very thin mica. The amounts of radiation of different wave-lengths coming through these three windows were measured by means of an x-ray spectrometer.

To determine the penetration of the radiation through the sheet of aluminum a number of measurements of the ionization current due to the radiation from the mercury vapor with and without the test sheet of aluminum in the path of the rays were made. The test sheet was placed close to the pin-hole, K, so that a negligible amount of the scattered radiation from it entered the ionization chamber.

In order to investigate the general or continuous spectrum radiation alone, without the line spectrum, it was necessary to apply to the tube a difference of potential insufficient to produce the line spectrum. It requires a difference of potential of 12,250 volts to produce the L_γ series lines of longest wave-lengths. Consequently, a difference of potential less than that has been used, namely: 11,780 volts. The short wave-length limit of an x-ray spectrum corresponding to this voltage is 1.05 Ångström.

Measurements of the penetration of the radiation through the test sheet of aluminum showed that 40% of the mercury radiation coming through the window, G, passed through the aluminum sheet and 38.8% of the radiation coming through the window, H, passed through the aluminum sheet. In order to find out whether or not the radiation is homogeneous, additional sheets of aluminum were placed in front of the windows and then the percentage of radiation passing through the test sheet was measured. The measurements showed for the mercury radiation coming through the window, G, that after this radiation had gone through one sheet of aluminum (weighing 0.02773 gr. per sq. cm.), 47.5% passed through the test sheet, and, after the radiation had gone through an additional sheet of aluminum (weighing 0.02717 gr. per sq. cm.), 51.3% passed through the test sheet. Similar measurements for the radiation coming through the window, H, showed that 46.1% of the radiation that had gone through the first sheet of aluminum passed through the test sheet, and that 50.7% of the radiation that had gone through the two sheets of aluminum passed through the test sheet. All these percentages have been corrected for the natural leak of the ionization chamber and for any small ionization currents that it detected when the mercury heater was not running. It appears, therefore, that the mercury radiation both in the direction of the electron stream and perpendicular to it becomes

more and more penetrating as it passes through successive layers of aluminum. In other words, neither beam of rays is homogeneous.

The mercury radiation through the window, *H*, appeared to be slightly more penetrating than that through the window, *G*. This, however, is due to the fact that the mica in window, *H*, was somewhat thinner than that in window, *G*, as will be seen later.

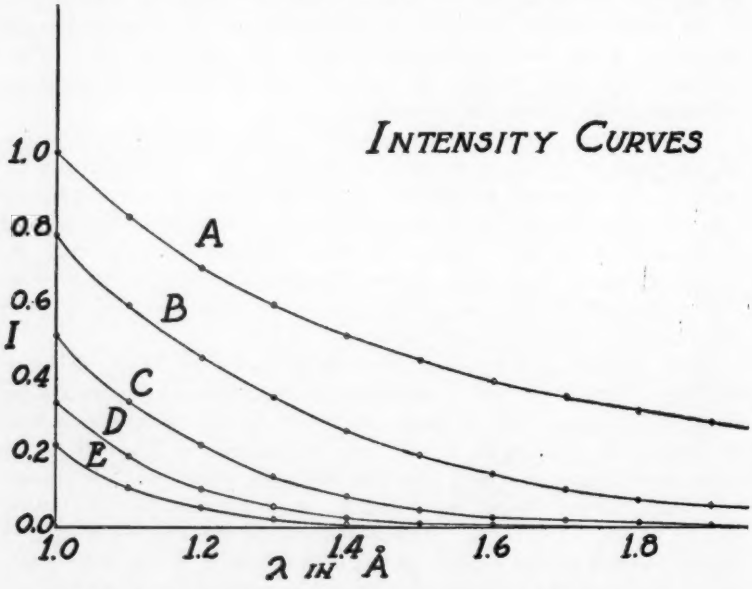
If the radiation has been homogeneous and had had a wave-length equal to that of the short wave-length limit of the spectrum, 1.05 Ångström, 61.3% of it would have passed through the test sheet of aluminum. This percentage has been determined by measuring the absorption of monochromatic radiation produced by reflection from a calcite crystal in an x-ray spectrometer. It appears, therefore, that the wave-lengths of the inhomogeneous beams of rays are on the average somewhat longer than that of the short wave-length limit of the x-ray spectrum. They are not very much longer, however, for the absorption of aluminum increases as the cube of the wave-length.

The intensity of the mercury radiation coming through the window, *H*, appeared to be five or six times as great as that coming through the window, *G*. It would be difficult to estimate accurately how much of this difference may be due to the fact that the beams came from different thicknesses of mercury vapor. We may conclude, however, that the intensities per mercury atom in the direction of motion of the electron and perpendicular to it are of the same order of magnitude. Further experiments on this point are in progress.

Some interesting conclusions have been drawn by D. L. Webster[2] and his students from their experiments on the general x-radiation coming from solid targets. One of these conclusions is that the radiation from an indefinitely thin target should have a maximum of intensity at the short wave-length limit and that this intensity should decrease beyond the limit, toward longer wave-lengths, approximately as the inverse square of the wave-length. Kramers[3] came to the conclusion that it would decrease almost exactly as the inverse square of the wave-length in his theory of the radiation that should be emitted by an electron approaching an atomic nucleus. He based his theory on classical ideas and Bohr's correspondence principle, assuming arbitrarily that the radiation would be cut off at the short wave-length limit given by the quantum equation.

In order to calculate the theoretical amount of radiation that should pass through the test sheet of aluminum, if the distribution in the spectrum follows the above inverse square law, it is necessary to determine accurately the absorption of x-rays of different wave-lengths by the mica windows and the sheets of aluminum. This has been done as described in the previous note by measuring the absorption of a few mono-

chromatic rays, produced in an x-ray spectrometer, and then calculating the percentages of radiant energy passing through the various sheets of material by means of the accurately known law of variation of their absorptions with the wave-length. This spectrometric method of measuring the penetration of the rays through the windows and sheets of aluminum gives more accurate results than the measurements of their masses per square centimeter, for it automatically corrects for small impurities. The curves in figure 3 give the results of these measurements and calculations for the end window, *H*. The abscissas represent the wave-lengths and the ordinates, the intensities expressed on an arbitrary scale. Corrections have been made in these curves for the absorption of the radia-



tion by the air through which it passed between the tube and the ionization chamber. The upper curve, *A*, shows the theoretical distribution of energy in the spectrum according to the inverse square of the wave-length law, starting with a short wave-length limit of 1 Å. Curve *B* represents the distribution of energy in the spectrum after the radiation has passed through the mica in the window, *H*, of the tube, through the air from the tube to the ionization chamber and through its mica window, *I*. Curve *C* represents the theoretical distribution of energy in the spectrum after this radiation has passed through the test sheet of aluminum in addition to the mica and air. The area under the curves

give the total intensity of the radiation in the respective cases. The area under curve, C, from $\lambda = 1.05$ on divided by the corresponding area under curve, B, represents therefore the fraction of the radiation that, theoretically, should pass through the test sheet of aluminum. The ratio of these areas shows that for the radiation coming through window, H, 38.2% should pass through the test sheet of aluminum. This theoretical percentage should be compared with the observed percentage 38.8%. Curves D and E have been drawn to represent the radiation that should come through the aluminum when the additional sheets are placed in the path of the rays. Corrections have been made for the slight differences of the masses per square centimeter of these sheets. Similar curves have been drawn for the radiation coming through the side window, G. All the curves were drawn on a large scale extending out to long wave-lengths and the areas under them determined with considerable precision. The numerical results of the calculations, as well as of the experimental measurements appear in table 1.

TABLE 1

PERCENTAGE THROUGH TEST SHEET

NUMBER OF SHEETS	SIDE RADIATION		END RADIATION	
	OBSERVED	CALCULATED	OBSERVED	CALCULATED
0	40.0%	38.4%	38.8%	38.2%
1	47.5	46.4	46.1	46.2
2	51.3	50.2	50.7	50.0

The first column contains the number of sheets of aluminum placed near the windows. The second and third columns contain the observed and theoretical percentages of the radiation passing through the test sheet of aluminum for the radiation projected at right angles to the stream of electrons. The fourth and fifth columns contain the same percentages for the radiation projected in the direction of motion of the electrons. It appears that the percentages calculated from the inverse square of the wave-length law agree very well with the observed values. The air corrections mentioned above have been made in the calculated values of the percentages as recorded in the table. The corrections were applied during the latter part of April. Previous to that the calculated values always came out several per cent smaller than the observed values. The radiation appeared to be actually somewhat more penetrating than would be expected according to the inverse square of the wave-length law. It had not been thought possible that the passage of the radiation through thicknesses of air ranging from 12 cm. to 22 cm. could so filter out the longer rays as to account for these differences. It was thought that the differences might be due to some excess radiation at or close to the short wave-length limit. The air corrections, however, brought the theoretical percentages so close to the experimental values that their differences may reasonably be regarded as due to experimental errors. Small corrections

such as those for the fact that a few of the electrons must have had velocities slightly less than the maximum and for the fact that not quite all of the radiation may have been absorbed in the ionization chamber (which contained methyliodide) would tend to decrease the calculated percentages and increase the amounts by which they fall below the observed values. The calculated and observed values, however, agree so well that we may conclude that, if there is any excess radiation at or close to the short wavelength limit, its intensity must be quite small.

After the experiments reported in the previous note (loc. cit.) had been finished, the tube was purposely broken and the absorption of the glass window determined for homogeneous rays in the x-ray spectrometer. Calculations similar to those described above show that in the earlier experiments, 56% of the radiation coming through the glass window theoretically should have passed through the test sheet of aluminum, whereas, as previously reported, 57.5% passed through it. The results of the earlier experiments, therefore, indicate that the theoretical penetration of the radiation is approximately that observed.

An attempt to obtain a spectrum of the mercury radiation is being made by means of a small spectrometer inside the tube.

[1] William Duane, "The Character of the General Radiation," *Proc. Nat. Acad. Sci.*, September, **1927**, p. 662.

[2] *Phys. Rev.*, March, **1923**, p. 323.

[3] *Phil. Mag.*, November, **1923**, p. 836.

ON THE QUANTITY OF ELECTRICITY DISCHARGED IN A LIGHTNING STROKE

By A. W. Simon

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The experimental field work of Norinder[1] on the extent of the electric fields of thunderclouds and magnitudes of these fields makes it possible to calculate approximately, or at least determine the order of magnitude of, the quantity of electricity involved in a lightning stroke.

If a gradient F is maintained over a level surface (the earth's surface) of area A, the quantity of electricity stored on the area is given by

$$Q = AF/4\pi. \qquad (1)$$

Norinder has established experimentally that the extent of the field of a thundercloud is of the order of magnitude of one circular kilometer and from the work of Peek[2] we may conclude that the maximum gradient

is around 3300 volts/cm.; inserting these values in the equation we obtain the corresponding Q as *2.3 coulombs*.

In place of a gradient of 3300 volts/cm. we might substitute 30,000 volts/cm. (the dielectric strength of air, below which the actual gradient must certainly be) and, using again 1 circular kilometer for the area, obtain as the corresponding quantity *21 coulombs*.

It is seen that equation (1) gives the maximum charge of a thin conducting disc placed above a conducting plane, provided we substitute for F the dielectric strength of air, since the gradient cannot assume a value greater than this without spark-over. It would correspond to storm conditions provided the whole charge of the cloud were concentrated in a thin layer at the base of the cloud.

The foregoing equation assumes a *surface* charge, let us next investigate a *volume* charge. Norinder has stated that in many cases the volume discharged is less than 500 meters in diameter, assuming again that 1 spherical kilometer represents the extent of the volume charge, we have as the maximum charge that can be stored in a sphere of radius R:

$$Q = R^2F, \tag{2}$$

where F is the gradient at the surface. Putting F = 30,000 volts/cm. in this equation and R = 500 meters, we have the corresponding quantity as *83.3 coulombs*. Of course this equation holds only provided the opposite charge is spread over a concentric conducting sphere or is infinitely far removed. In practice the quantity actually stored must always be less, since the opposite charge will be concentrated on *one* side of the sphere, that is on the earth's surface.

If a charge, Q, is uniformly distributed throughout the volume of a sphere of radius, R, with its center at a height, H, above a level conducting earth's surface, we have the maximum gradient at the earth's surface given by the equation:

$$F_S = \frac{2Q}{H^2}. \tag{3}$$

If we substitute the values F_S = 3300 (Peek, Norinder) and H = 1.5 km., which values correspond to a sphere 1 km. in diameter with its lower pole 1 km. above the earth's surface, we have the corresponding quantity as *41.3 coulombs*.

If for this same case F_P is the gradient at the lower pole of the sphere we have:

$$F_P = Q\left[\frac{1}{R^2} + \frac{1}{2(H-R)^2}\right], \tag{4}$$

whence for F_P = 30,000 volts/cm., we have the corresponding quantity as *80 coulombs*.

It is interesting to combine (3) and (4) as follows: (1) Put for F_P the dielectric strength of air and calculate the corresponding value of Q. (2) Substitute this value in (4) and calculate F_S. This would give the maximum gradient at the earth's surface for this special case. If this is done, we obtain as the corresponding gradient F_S the value 6410 volts/cm., which checks approximately with the gradient which Norinder surmises exists at the point where the lightning stroke occurs just previous to the stroke.

From the values given above it seems very probable that the quantity of electricity discharged in a lightning flash is of the order of magnitude of 10 coulombs. Very likely the value 2.3 coulombs is somewhat low while all the remaining values are much too high.

With regard, however, to equation (1) it is to be noted that if the gradient F were everywhere known over a given area, A, of the earth's surface, the corresponding quantity Q stored on the area would be given directly by

$$Q = \int F dA/4\pi \qquad (5)$$

so that if it be true that the maximum gradient at the earth's surface cannot exceed 3300 volts/cm. and that the area discharged is one circular kilometer, then it follows that the order of magnitude of the quantity discharged in a stroke is less than 10 coulombs, rather of the order of 1 coulomb.

[1] *Electrical World*, **83**, 223 (1924).
[2] *Jrl. Frank. Inst.*, **199**, 142 (1925).

SECONDARY ELECTRONS FROM COBALT

By Myrl N. Davis

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Communicated May 7, 1928

Numerous investigators have studied the emission of electrons from metal surfaces under bombardment by slow electrons. When this emission is studied as a function of the energy of the primary electrons, results characteristic of the metals under investigation have been found.

Farnsworth[1,2,3] and Petry,[4] working with iron, nickel and copper, have found particularly characteristic results for those metals. With this fact in mind, it was decided to study the secondary emission from cobalt for low velocity primary electrons. The sequence of iron, cobalt, nickel and copper in the atomic series, and the ferro-magnetic character of the first three of the above-named metals gave rise to the hope that

some interpretable variation from metal to metal, or some characteristic similarity might be discovered.

Apparatus and Procedure.—Two different vacuum tubes have been employed in this investigation. The first was one used by Farnsworth in a portion of his work on iron, nickel and copper. A careful description of this tube is included in his reports.[2,3] A beam of primary electrons was secured by accelerating thermions from an oxide-coated plate through a series of diaphragms. The primary current was measured by means of a Faraday cylinder. The target, which could be moved by a magnetic control, was then interposed in the path of the beam and the current to it was measured, the secondary electrons going to a cylinder located directly in front of the target. The difference between the currents to the Faraday cylinder and to the target gave the secondary current. When not in the beam of primary electrons, the target was withdrawn into a side tube. In this position it could be heated by high frequency induction for outgassing.

The second tube, which will be referred to as tube II in the remainder of the report, differed essentially from the tube just described only in the elimination of the Faraday cylinder for measuring the primary current. The target was fixed in position in the beam of primary electrons. The secondaries were again received in a cylinder in front of the target, the current to this cylinder giving the secondary current. The sum of the currents to the target and to the cylinder in front of the target gave the primary current. In this tube the primary electrons were accelerated from the middle portion of a single loop tungsten filament. The target was heated for outgassing by electronic bombardment, a second filament acting as a source for these electrons.

The targets used in this work were ground from a sample of cobalt secured through Professor L. R. Ingersoll from the Deloro Smelting and Refining Co., Deloro, Ont. Chemical analysis showed the composition to be Co 98.67; Ni 0.44; Fe 0.60; Al 0.05; S 0.06 and Si 0.00 per cent.

After a vacuum has been obtained, the tube was always baked until the pressures were about 10^{-6} mm. Hg with the furnaces hot. The targets were then heated to a bright yellow until further heating caused no change in the characteristics. At the end of this outgassing process, the pressures were about 5×10^{-6} mm. Hg with the target hot. The pumps were left running continuously while the data were being taken, the pressures being so low that the mercury stuck to the top of the capillary in the McLeod gauge when the pressure was being measured.

Results.—The data obtained with either tube were reduced by taking the ratio of secondary to primary current for each accelerating potential impressed on the primary beam. This gave the number of secondaries per primary electron and made it unnecessary to keep the primary current

constant throughout a run. The ratios obtained were then plotted against the corresponding accelerating potentials.

The curves in figure 1 will serve as a basis for discussion of the results of the investigation. Curve I is typical of all the data obtained with tube I after the target had been outgassed, and of results with tube II after certain heat treatments, while Curve II was only obtained with tube II after a different heat treatment. Curves like II were obtained after the target had been heated for a total of fifty-four hours at temperatures ranging up to a bright yellow. The latter temperatures had been maintained for about ten hours. Six hours further heating at the same tempera-

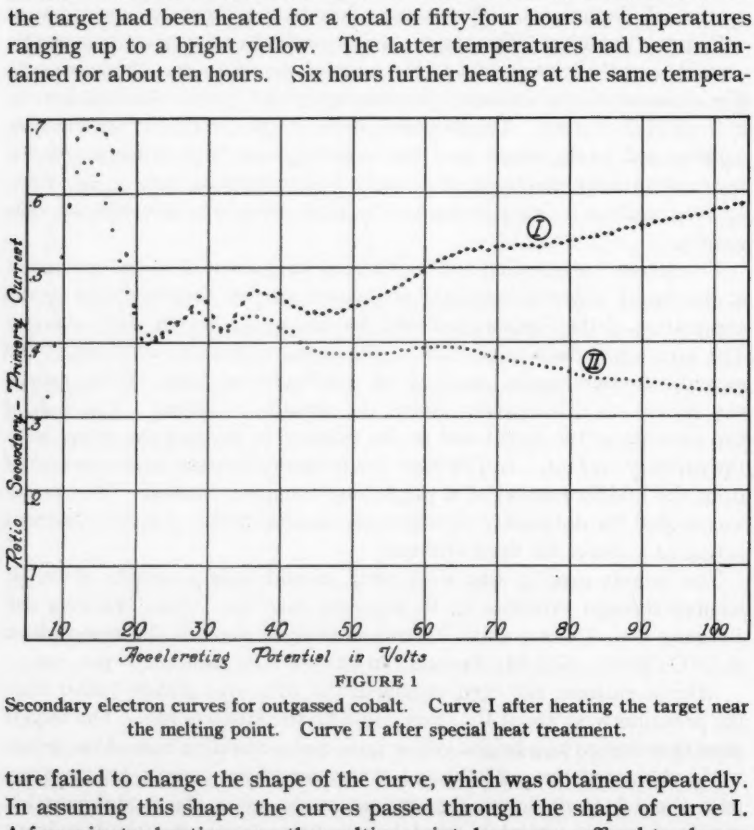

FIGURE 1

Secondary electron curves for outgassed cobalt. Curve I after heating the target near the melting point. Curve II after special heat treatment.

ture failed to change the shape of the curve, which was obtained repeatedly. In assuming this shape, the curves passed through the shape of curve I. A few minutes heating near the melting point, however, sufficed to change conditions in such a way that curves of type I were again obtained. During this heating, evaporation of the target took place very rapidly. Further heating at a yellow heat failed to cause a reversion of the curves obtained to the shape of II.

The values of the abscissas in the curves have been corrected by the addition of 4.9 volts, the work function of the tungsten filament, to the

observed values for the accelerating potential. As plotted, the abscissas give the energy of the electrons after being accelerated by the work function of the target. No correction is necessary for the potential drop in the filament used as source of electrons, since this was negligibly small over the part of the filament used, and since the accelerating potentials were applied through a potentiometer arrangement.

Before the target had been heated for outgassing, the secondary emission curve was always low and without marked maxima and minima. As the process of outgassing proceeded, the various maxima and minima began to develop, and the values of the secondary emission began to increase in magnitude. Only as the outgassing appeared to near completion did the first maximum attain its greatest height and the minimum at 45 volts make a definite appearance. After the maxima and minima had once appeared there was no change in their position. Only their magnitudes changed thereafter. When the tube was allowed to stand for some hours without heating of the target, the height of the curve usually changed only slightly. A few seconds heating was always sufficient to bring back the previously observed values.

Discussion.—The secondary emission from cobalt is much greater than that from any other metal as yet investigated. Not only are the maxima much more pronounced, but even at the minima the height of the curve is decidedly above the corresponding values for other metals. Plotted to a scale in which the ordinates are large, iron and nickel show something of the same form of curve, but for them the breaks are not so pronounced, being at lower values of the primary energy, and do not extend to as high values of the primary energy. If one assumes that the first maxima of the curves for these three metals "correspond," it then turns out that these maxima shift toward higher primary energies with (and in the order of) increasing atomic weight, rather than, as might be expected, atomic number. Curves given by Farnsworth[3] show first maxima at 1.5 and 3 volts for iron and nickel, respectively. These values have been corrected for contact potential, but require a further correction for the work function of the target before comparison with the value 14 volts which was found for cobalt, but this correction cannot be expected to invert the order of these two metals.

The crystal lattice of cobalt is known to change from hexagonal close-packed to face centered cubic at 470°C., to maintain this form throughout the range of temperatures to 1120°C., and to change to hexagonal close-packed once more at the latter temperature. Both hexagonal close-packed and face centered cubic forms of lattice have been observed at room temperature. Previous to the time during which curves of type II were obtained, the target had been subjected to a long period of heating in the range of temperatures in which the hot target should have had the

face centered cubic structure, and this type of curve continued to be observed as long as heating at this temperature was continued. It may be that conditions were correct for the target to carry down some of the cubic form to low temperatures, and that the difference between curves I and II is due to a difference in crystal structure of the target. The exact conditions necessary to bring about this result have not yet been determined.

Krefft, in a paper which has recently appeared,[5] describes an investigation of the secondary emission from tungsten. He was able to observe with the target hot as well as with it cold. He finds that his data with a cold target must be taken within a few minutes after the target has been heated in order to obtain the same results for a cold target as for a hot. Standing cold for half an hour was sufficient to cause a decided change in the form of the curve, particularly that of the first maximum. This change he ascribes to the adsorption of gas by the surface of the target. As has already been remarked, no such rapid change was observable in this work after a sufficient heat treatment. Even if the target were allowed to stand without heating for more than thirty hours, the only change observable was a slight lowering in height of the whole curve. We may conclude, therefore, that either (*a*) the cobalt surfaces were more thoroughly outgassed and picked up gas more slowly than the tungsten, or (*b*) the cobalt was *never* free from gas, and on this account changed less rapidly upon standing. The latter conclusion seems hardly consistent with the general trend during outgassing, and especially with the fact that the results were unaffected by heating very nearly to the melting point. We are, therefore, inclined to accept alternative (*a*) and to argue that the cobalt maxima and all of Krefft's maxima except perhaps the one which developed on standing cold are characteristic of the metals themselves. The effect of observing with a hot target will be studied in the near future. Krefft suggests that the maximum which appears at 15.5 volts may be due to the ionization of a layer of oxygen molecules on the surface of the target. This maximum may be compared with maxima for cobalt appearing at 9.1 and 24 volts (the uncorrected values should here be used). These values are too low and too high, respectively, to be attributable to ionization of the common gases.

Krefft gives some evidence of coincidence between values of breaks in the secondary emission curves for carefully outgassed tungsten with critical potential values for that metal. It was thought at the conclusion of the work with tube I in this investigation that similar evidence was offered by the many small breaks in the cobalt curves. Further work with tube II does not seem to support this idea, although some further study of this point will be made. A single run, covering the range from 3 to 40 volts at $^1/_{10}$ volt intervals was made. Conditions throughout the run were

very steady, and no evidence of the existence of breaks corresponding to critical potentials was obtained. Unhappily, the target was melted by excessive heating before further observations of this character could be performed.

In conclusion, the author wishes to extend his thanks to Prof. C. E. Mendenhall, whose suggestions and coöperation made this work possible.

[1] H. E. Farnsworth, *Phys. Rev.*, **25**, 41 (1925).
[2] H. E. Farnsworth, *Ibid.*, **27**, 413 (1926).
[3] H. E. Farnsworth, *Ibid.*, **31**, 405 (1928).
[4] R. L. Petry, *Ibid.*, **26**, 346 (1925).
[5] Hermann E. Krefft, *Ibid.*, **31**, 199 (1928).

SPARKS OF THE INDUCTION COIL BETWEEN MUCRONATE ELECTRODES

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1. *Spark Gap $x = 2$ Cm.*—The coil of the usual pattern with platinum spring break. From a single storage cell it produced sparks a little over 3 cm. between points under best conditions. The pitch of the break could be varied by tension from *G* to *c* judging by the ear. The secondary was connected with the mucronate electrodes, figure 1, having a spark gap from $x = 1$ to over 4 cm. and the needles y,y', both of the positive and negative plates E,E' of the gap, were actuated for protrusion y, with micrometer screws, S,S'. The hard rubber base *B* supports the brass posts P,P'.

The sparks were all white as a rule with exceptions to be noted. There were no purple cones. As it is convenient to introduce a terminology here, I shall call the promiscuous spark discharge from edge to edge of the electrodes EE', a spark rain. This does not depend on incidental conditions of the surface of the disc electrodes; for if the sparks are between the top edges, they remain so when either disc is rotated on its axis. They show a marked tendency to remain near the top even when the plates are slightly oblique or when the spark gap is inverted. A wind blown across the spark gap is usually ineffective.

Darts designate the nearly linear sparks which pass from the point of the needle to the opposite plate. Sometimes they contain purple strands and they may be purpled by special interference. All such darts terminate in the darkish space already described, and this was here usually nearer the negative plate.

With these preliminaries a random example ($x = 2$ cm.) of the observed spark phenomena may be summarized. Beginning with the anode, $y = 0$ to 0.08 cm., the white spark rain occurs, darts beginning at $y = 0.06$ to 0.08 cm. usually, so that the positive electrode emits darts when the negative electrode heretofore emitted the wind. Furthermore, $y = 0.08$ to 0.30 cm. marks the occurrence of vigorous darts; but they are extinguished by the negative body (charged hard rubber rod) acting from a distance of 20 cm. or more, on any side. At $y = 0.3$ to 0.6 and above, the negative body is *without* influence on the darts from any side or distance. Darts produced at $y = 0.5$ cm. were retained (owing to ionization), with y decreasing to 0.16 cm.

Cathode.—The behavior is here quite different; for from $y = 0$ to 0.40 cm. the white spark rain remains. If the negative body is suddenly jerked away after having extinguished the sparks, an accentuated spark rain often reappears, momentarily. At $y = 0.40$ to 0.6 cm. the spark rain condenses to a white, nearly linear dart. The negative body is now quite inactive.

2. *Larger x-Values.*—The negative body, as a rule, was able to quench the darts at their inception, i.e., at the lowest y at which they are producible. The effect is further facilitated by lowering the frequency and thus further reducing the vigor of the sparks. Anode darts were, as a rule, much more subject to the negative body than those coming from a cathode needle. A resonant effect of pitch of spring break was not observed. High pitch obviously intensifies the spark.

The cathode darts often begin profusely divergent or casually. The true and shorter cathode dart, however, is straight and starts out sonorously almost with a roar, so that it is then easily recognized. Unfortunately, this is not always the case. Its sparking distance is but 0.6 of the anode sparking distance. The dark spaces of the darts seen in the telescope are purple filaments, often with traces of the white darts from which they degenerate intermixed.

A systematic account of the results obtained cannot be given here, except insofar as the summarized results for minimum sparking distances were

$x = 3.0$ cm.	cathode $x-y = 2.0$ cm. darts	anode $x-y = \ldots$ darts
3.6 cm.	2.2 cm.	3.5 cm.
4.4 cm.	2.3 cm.	3.6 cm.

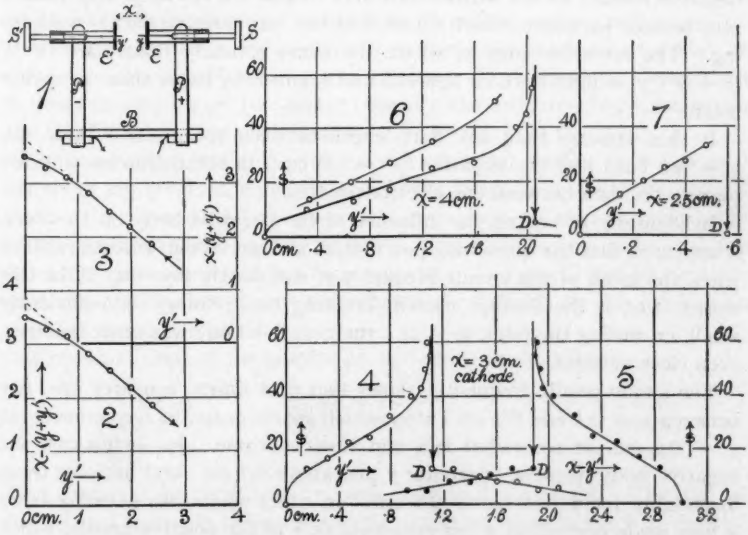
As the set is certain to within 0.04 cm., the differences here reflect real differences of potential due to the functioning of the spring break. Moreover, in these experiments the true straight cathode dart was a definite occurrence throughout. When this is not the case and the integration of

divergent spark filaments is the only criterion, y may vary one or two millimeters.

3. *Stray Positive Convection Currents.*—The curious presence of initial spark rain ($y = y' = 0$) between the upper edges of the E,E' plates in figure 1 has been referred to. Trying this experiment in the dark, I noticed that purple positive radiation issued from rough parts (screws) behind E and entered the spark gap. Cutting off this radiation by a rubber sheath or hard rubber disc, much of the spark rain vanished and the cathode dart appeared from darkness at the proper value of y, and straight. The cathode y value could now be sharply measured, as for instance:

x = 2 CM.	y = 0.24 CM.	$x-y$ = 1.76 CM.
2.5	0.70	1.80
3.0	1.28	1.72
4.0	2.30	1.70

The set being definite to 0.04 cm., the differences are actual. Unfortunately, when the sparks are intensified by raising pitch the spark rain reappears with the vagueness as to set. The same is true of the anode where there is mere intensification as y increases.



The darts observed in the telescope near the cathode needle sometimes show a crinkly or a right-angled zig-zag structure, consisting therefore of short line elements, not colinear. Degeneration into purple threads is also seen. As the darts are each instantaneous, this cannot be ascribed to motion of the telescope and they are continuous near the anode needle.

4. *Darts between Needles.*—The cathode dart has been found to be constant in length for all available distances, x, at the same potential difference. The same is true for the longer anode dart, though here the discrimination as to a correct set is less definite.* The question arises, therefore, whether in passing continuously from the cathode dart to the anode dart in a sufficiently wide gap, x, between plates, the dart will grow continually longer. This is the case.

The experiment is made by using cathode and anode needles together, the numerical protrusions being y' and y, respectively. Hence, the length of the dart is $x - (y + y')$. Beginning with $y' = 0$, y' and y are increased in steps, successively, and the dart length found for each paired values of y and y'. Figures 2 and 3 are examples of such results, the dart length $x - (y + y')$ being plotted against y' (cathode extrusions) as abscissas.

In an earlier series the mean rate of dart-length increase with the protrusion of cathode needle was $(x - (y + y')/y' = 0.77$, while the ratios of anode protrusion were $y/y' = 0.22$ only. The relations here were appreciably linear, the first point for $y = 0$ being uncertain. Figures 2 and 3 give results obtained in successively protruding and withdrawing the negative needle, as the arrows indicate. These curves as y' approaches zero become parabolic, which shows that the long positive dart is weakening. The rates for large y, where the curve is nearly linear, are $(x - (y + y')/y' = 0.82$, both for figures 2 and 3, naturally larger than the earlier mean.

It thus appears than any dart length between the positive (here say $y = 3.5$ cm.) and the negative ($y' = 1.9$ cm.) is obtainable by suitably placing the dart between the electrode plates.

Incidentally in testing the influence of the negative body on the dart, it appeared that the quenching power from at least 30 cm. was guaranteed when the pitch of the circuit breaker was sufficiently low (say F in this work); i.e., if the average current entering the primary is sufficiently small, on raising the pitch to A or c the negative body was quite inactive, even close at hand.

The anode needle frequently shows two dart limits, a nearer one, say between $y = 0.1$ and 0.2 cm., after which sparks cease, to recommence at $y = 0.4$ cm. or somewhat less and then continue. In such a case the negative body quenches darts for y just above 0.1 cm., and initiates them formidably for y just below 0.4 cm. In other words, the negative body is here again equivalent to an increment of y of the positive needle, which then virtually enters a more negative part of the field.

5. *Cathode Electric Wind of the Induction Coil.*—Hitherto my experiments have practically failed; but with the information obtained on the behavior of the cathode needle, a new attempt is suggested. The apparatus (Fig. 1) is available if one needle and screw are removed and the end of the tube

stem near S joined to the interferometer U-gage. Hence $y = 0$ throughout whereas y' the protrusion of the cathode needle is varied.

The results are given in figure 4, for a distance between plates of $x = 3.0$ cm., where the wind pressures (s roughly in half 10^{-6} atm.) at the anode plate are plotted against the protrusion y' of the cathode needle. At $y' = 0$ there is no needle point and no pressure; but this starts out rather abruptly at small y', then proceeds nearly linearly until near the sparking distance $y' = 1.1$ cm., where a very rapid acceleration of pressure is in evidence. With the occurrence of the dart at D, $x - y = 1.9$ cm., the pressure s sinks at once though the darts are still accompanied by a relatively weak electric wind. The sparking distance thus obtained is about the same as above. In figure 5, the gap between needle point and anode plate, $x - y'$ is made the abscissa, so that the figure shows the critical sparking distance $x - y' = 1.9$ cm.

Figure 4 is similar in general character to those found with the electric machine. The very low pressure here ($s = 60$) as compared with the large values in the early work ($s = 500$) in spite of the great voltages in the former case, are referable to the intermittance.

The endeavor was now made to work at higher pitch; but this proved fruitless for the incessant spark rain kept the wind pressures within $s = 10$. It was curious to note the occurrence of darts and spark rain together in these cases, the sparking distance of the latter (between plates) being even 3 times the length of the darts. Usually the negative body condenses the rain filaments to a single dart.

Similar experiments at greater ($x = 4$ cm.) and smaller ($x = 2.5$ cm.) distances are given in figures 6 and 7, the latter in a double horizontal scale. Very low pitch was necessary even here to obviate spark rain which at once reduces s nearly to zero. Two examples are given in figure 6 but the upper one could not be completed. In figure 7 this acceleration does not appear, but it was correspondingly difficult at this small x to establish spark-free winds.

The initial slope of the graphs $\Delta s/\Delta y'$ came out as follows:

$x = 2.5$ cm.	$\Delta s/\Delta y' = 5.4$	$x - y' = 1.9$ cm.
$x = 3.0$ cm.	2.9	1.9 cm.
$x = 4.0$ cm.	1.7–2.2	1.9 cm.

The sparking distance $x = y'$ was practically constant. The fall of $\Delta s/\Delta y$ as x increases indicates an attempt in all cases to reach a definite wind pressure just before the dart appears.

* Unless an initial y-interval of darts appears.

THE ADSORPTION KINETICS FOR MOLECULES ATTACHED AT MORE THAN ONE POINT

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Communicated May 1, 1928

The extremely short range of atomic forces limits the effectiveness of adsorptive forces to a region within an atomic diameter of the surface.[1] Furthermore, the portions of this force region directly overlying the surface atoms would be the seat of forces enormously more intense than the regions in between these atoms. Such a structure of the adsorptive forces leads to the conception that the separate atoms of the adsorbed molecule are attached to the separate atoms of the surface. Insofar as the geometric arrangement of the relevant atoms does not permit this, the adsorbed molecule will be stretched, an action which must play a part in catalysis.[2]

Recent experiments[3] of Beebe, Taylor and others on "differential" heats of adsorption show that when some gases are adsorbed on certain surfaces, less heat is evolved for the first portions of gas adsorbed than for later portions, indicating that the heat of adsorption is not the only factor

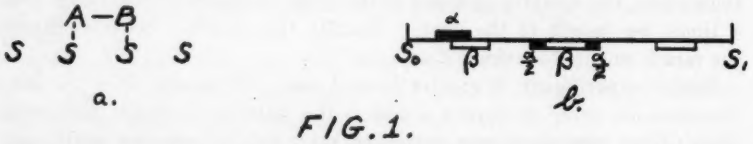

FIG. 1.

determining the life of an adsorbed molecule on the surface. In other of the experiments, the molecules first adsorbed did evolve more heat than the later ones.

From the above remarks, where it was concluded that the atoms of an adsorbed molecule should be considered to be separately attached to individual atoms of the surface, a certain kinetic complexity arises in the process of desorption which offers a possible explanation of these interesting results on "differential" heats of adsorption in the case of molecules with more than one atom.

Consider a molecule, $A—B$, adsorbed on a surface, the atoms of which are represented by S in the figure 1*a*. If, in desorption, the bonds are broken by separate elementary acts, the possibility is that unless both the $A—S$ and the $B—S$ bonds are broken within a very short time, the atom first knocked loose will have an opportunity of becoming attached again, so that if this time interval is short enough, molecules of this type could leave the surface less often than those attached by a single bond involving

an amount of energy greater than the sum of the heats of linkage for both the A—S and B—S bonds.

Let us represent a second of time by the straight line S_0S_1 (Fig. 1*b*). When A is knocked loose, let the average time it is loose before returning to the surface be α, the corresponding time for B being β. Let the number of vibrations of the underlying surface atoms (all alike) per second be M. If the distribution of energy among these vibrations be Maxwellian, $e^{-E_A/RT}$ would be the probability of a given vibration breaking the A—S bond and $Me^{-E_A/RT}$ would be the approximate number of times per second each surface atom would vibrate with sufficient energy to break the A—S bond, E_A being the energy necessary to break the bond. The corresponding number for the B atom would be $Me^{-E_B/RT}$.

Now let us lay off upon the line representing a second $Me^{-E_B/RT}$ segments each of length β. Then if we lay off, arbitrarily, a segment α in length, the chance that it has a point in common with a β segment (the condition assumed to be necessary for desorption) would be the same as the chance of a point falling within one of the latter segments if each end of each of these were enlarged by $\frac{\alpha}{2}$. This probability would be the ratio of the sum of the enlarged segments to the length representing a second, or $Me^{-E_B/RT}(\alpha + \beta)$.

If, when the A—B-molecule leaves the surface, it is instantly replaced, one would then expect $M^2e^{-(E_A+E)/RT}(\alpha + \beta)$ molecules to leave the pair of surface atoms per second, this being the number of α segments having a point in common with a β segment. If there are a great many (P) molecules attached to the surface in this way, the rate at which they would be desorbed per second would then be $PM^2e^{-(E_A + E_B)/RT}(\alpha + \beta)$.

If the molecules were attached by a single point, and this bond involved energy $(E_A + E_B) = E_T$, the rate at which they would come off would be $2PMe^{-E_T/RT}$. In deriving the last expression, as before, the probability of a given vibration breaking a given bond would be $e^{-E_T/RT}$, and the number of times a given molecule would be desorbed each second (if the molecules are instantly replaced each time) would be $Me^{-E_T/RT}$. Since the surface could now accommodate $2P$ molecules, the rate of desorption would be $2PMe^{-E_T/RT}$.

The singly and doubly attached molecules would thus come off at equal rates if $M(\alpha + \beta) = 2$, otherwise not. If the doubly attached molecules were desorbed by a single extremely violent vibration at one of the atoms, the necessary energy of the act must exceed $E_A + E_B$ because some energy would be used up in stretching the A—B bond.

Such a kinetic complexity may also have something to do with the alleged rough proportionality between the association of substances and their boiling points.

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[2] *J. Phys. Chem.*, **30**, 1134 (1926).

[3] Beebe, *Ibid.*, **30**, 1538 (1926); Kistiakowsky, *Proc. Nat. Acad. Sci.*, **13**, 1 (1927); Taylor and Kistiakowsky, *Z. physik. Chem.*, **125**, 341 (1927); Kistiakowsky, Flosdorf and Taylor, *J. Am. Chem. Soc.*, **49**, 2200 (1927); Garner, *J. Chem. Soc.*, 2451 (1927); Magnus and Kalberer, *Zeit. anorg. allgem. Chem.*, **164**, 345 (1927).

CHARACTERISTICS OF HOMOGENEOUS, EXOTHERMIC GAS REACTIONS[1]

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Communicated April 17, 1928

Previous investigations have shown that such gases as methane and hydrogen react slowly with oxygen in vessels of glass, silica or porcelain at temperatures in the neighborhood of 500°C. These reactions, if purely homogeneous, would be of considerable interest in their relation to gaseous combustion processes in general. However, the difficulty of obtaining reproducible measurements of reaction rate indicates that catalytic action by the walls of the reaction vessel is to be taken into consideration. Indeed, it has been questioned whether any homogeneous reaction whatever occurs under these conditions. In order to obtain evidence on this point, and if possible to isolate the homogeneous reaction, we have recently carried out experiments in which the suppression of the wall reaction was attempted. Our results indicate some success in this regard. Considerable decreases in reaction rate have been effected. In addition we have obtained results which seem to necessitate the predominance of a homogeneous gas reaction under certain circumstances, and which reveal something of its nature. We find that the rates of these oxidation reactions may be greater in an empty reaction tube of Pyrex glass than in a tube filled with broken Pyrex, although the latter should favor the surface reaction. Further, this result is the more pronounced after poisoning. We desire to cite some of the evidence which leads to these conclusions, and to discuss briefly its implications.

The activity of a contact catalyst can usually be diminished by exposure to a substance which adheres to its surface, and which is itself inert. For the present purpose we chose to try potassium chloride. The first experiments were made on 50 per cent methane-oxygen mixtures by the flow method. A reaction tube of Pyrex was made up, washed out with nitric acid and water, and tested. The tube was then rinsed out first with a

dilute potassium chloride solution and then several times with water, and again tested. At 650°C., and 3 seconds contact time, 40 per cent of the methane was oxidized before this treatment, and only 5 per cent after. We next determined whether an increase of poisoned surface had an influence on the rate, in order to judge of the completeness of the suppression of the surface reaction. Another reaction bulb was made up, filled with broken Pyrex, treated with potassium chloride solution, and tested. This bulb was considerably larger than the other. The contact time was twenty seconds instead of three. It was found that even with the 6-fold increase in contact time only 10 per cent of the methane was oxidized. This result clearly indicated that the glass packing inhibited the reaction—certainly that no positive catalytic action predominated. We conclude that the reaction which is suppressed by the packing is homogeneous.

Subsequent experiments have shown that the potassium chloride treatment is not essential to the inhibitory action. Clean glass fragments have a similar, though less marked, effect. Further, this type of behavior is not limited to the methane-oxygen reaction nor to oxidation reactions. Evidence of inhibition by glass packing has been obtained in the oxidation of hydrogen, and of isobutane, and in the condensation of acetylene and of ethylene. With respect to the hydrogen-oxygen reaction, our conclusions are strengthened by experiments recently published by Hinshelwood and Thompson.[4] These experiments were carried out statically in reaction vessels of silica. It was found that, at the lower temperatures, reaction was more rapid in the presence of a packing of silica than in an empty bulb, but at higher temperatures the reverse was true. We have been able to obtain considerably lower rates than Hinshelwood and Thompson by suppressing the surface reaction, and, so far as our experiments have gone, we find that the reaction rate in the packed tube is always less than that in the unpacked tube. Some of our data are given in the following table. The percentage of hydrogen converted to water in packed and empty bulbs at three temperatures are given. A 50 per cent hydrogen-oxygen mixture was used. The contact time was 20 seconds and was identical in the two bulbs, packed and unpacked. Attention is especially directed to the rapid increase in the ratio of rates in the two bulbs.

HYDROGEN-OXYGEN COMBINATION

TEMP., °C.	PER CENT CONVERSION UNPACKED TUBE	PACKED TUBE	RATIO
525	0.08	0.06	1
550	0.19	0.10	2
575	18.1	0.20	90

It is probably a general rule that when surface catalysis accompanies a homogeneous gas reaction, the surface action is more in evidence at the

lower temperatures. Thus, H. A. Taylor[5] has shown that in the decomposition of hydrogen iodide the ratio of the rate in a packed Pyrex tube to that in an empty tube is nearly 6 at 400°C. but only 2.5 at 520°C. As the temperature is raised, the homogeneous reaction comprises a larger fraction of the total, so that the rates in packed and unpacked bulbs approximate to the same value. This doubtless accounts also for the low temperature results of Hinshelwood and Thompson. But, in the reactions we have under review, the rate in the unpacked bulb may far exceed that in the packed bulb. It is clear that some new factor must come into prominence.

This is, we believe, the development of reaction centers in the body of the gas. These reactions are all exothermic. Newly formed molecules of products possess not only the energy subsequently to be liberated as reaction heat but also the original heat of activation of the reacting molecules. When such molecules are produced within the gas the effect is the equivalent of an extremely high local temperature; but it is meaningless to speak of temperature in this connection, since the scale is a molecular one. The energy locally available is more than enough to activate a single set of reactant molecules, since the original energy of activation is alone sufficient for this purpose. In fact, the accumulation of energy probably leads to the development of what may be described as a miniature explosion wave, as if the gas had been ignited at the original reaction center. As such waves develop, they must, if unhindered, overlap and produce thereby a cumulative effect.

Completely adiabatic conditions would be ideal for such a development. Actually, however, contact with the relatively cold walls of the reaction vessel must bring this action to an end. The energy of the reaction centers is absorbed, and distributed to the surroundings, or in smaller units to more normal molecules of the gas. Limitation of development of reaction centers is more efficient when the tube is filled with broken glass because the volume available for development is decreased. The bits of glass act as temporary reservoirs for the excessive energy of reaction centers.

The cumulative effects of the development of reaction centers will be most apparent when the centers are most numerous. A part of the energy developed in a center is doubtless lost to it by conduction, but such portions of energy may be adopted and utilized by other centers in close proximity. This amounts to auto-acceleration; and if the rate of formation of centers is related to concentration and temperature by the conventional functions for reaction rate, it should follow that auto-acceleration is most apparent at higher concentrations and temperatures. This would explain the high-order reaction found by Hinshelwood and Thompson in the hydrogen-oxygen combination at higher temperature, and the rapid

increase in rate with increasing temperature which we have observed in empty tubes.

The above provides a working hypothesis for the study of homogeneous, exothermic gas reactions. Whether or not additional hypotheses (such as that peroxide formation plays a rôle in oxidation reactions) must be provided, further study will reveal. We believe, however, that the formation and development of reaction centers with which are associated excessive amounts of energy must be of first importance.

[1] This paper contains results obtained in an investigation carried out as part of Project No. 7 of American Petroleum Institute Research. Financial assistance in this work has been received from a research fund of the American Petroleum Institute donated by Mr. John D. Rockefeller. This fund is being administered by the Institute with the cooperation of the Central Petroleum Committee of the National Research Council. Professor H. S. Taylor of Princeton University is Director of Project No. 7.

[2] Research Associate, American Petroleum Institute.

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[4] Hinshelwood and Thompson, *Proc. Roy. Soc.*, **118A**, 170 (1928).

[5] H. A. Taylor, *J. Phys. Chem.*, **28**, 984 (1924).

ON THE MECHANISM OF CHROMOSOME BEHAVIOR IN MALE AND FEMALE DROSOPHILA

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The singular fact that the interchange of genes between a chromosome and its mate takes place only in the female of Drosophila offers a puzzle, the key to which is unknown. A clue is said by some to be furnished by the fact that this crossing-over takes place in the homogametic sex and may therefore be associated with the difference in the behavior of the sex chromosomes in the oöcytes and spermatocytes. This view is supported by the fact that in certain of the Lepidoptera where the homogametic sex is the male, the male is the sex to show crossing-over. On the other hand, rats show gene interchange between homologous chromosomes in either sex. The problem is thus open. It might be thought that were it possible through some agency to cause crossing-over in the male of Drosophila the processes through which the chromosomes pass might thus be made comparable to those of the female or vice versa if the female could be made to have complete linkage of the genes within its homologous chromosomes the mechanism could be interpreted as the same as that of the present male.

It will be recalled that in 1922[1] data were presented to show that the linkage relations of the female Drosophila could be changed from the normal percentages to complete linkage by the action of a recessive factor in the third chromosome. The influence of this factor extended to all chromosomes. Is the behavior of the chromosomes of such females like that of the males in the gonia and auxocytes? This question can be answered in the negative, the mold through which the chromosomes pass in the female is different from that of the male even where the female showed complete linkage. This is shown clearly in outcrosses of males and females homozygous for the factor for complete linkage and producing a high frequency of triploids, sex intergrades, non-disjunctional males and females, and a class of weak flies showing eyes of a fine ommatidial pattern, and slender, short bristles.

Table 1 shows the results of this cross. Where the cross is made with the male carrying the suppressor for crossing-over the progeny resulting were 862 males and 899 females. When the female is the animal carrying

TABLE 1

$$♀\ \frac{Bx}{+}\ \frac{Cy}{+}\ \frac{D}{+} \times ♂\ \frac{+}{+}\ \frac{+}{+}\ \frac{cx}{cx}$$

CULTURE NO.	♂	♀
9006	147	135
9007	121	185
9032	86	67
9035	53	57
9040	75	79
9044	66	46
9102	15	22
9111	159	160
9134	140	148
	862	899

$$♀\ \frac{+}{+}\ \frac{+}{+}\ \frac{cx}{cx} \times ♂\ \frac{Bx}{-}\ \frac{Cy}{+}\ \frac{D}{+}$$

CULTURE	+♂	BX♀	♂	♀	TRIPLOID	BX♂	+♀	SLENDER BRISTLES
9033	53	43	2	1		12	13	1
9039	46	49	1	2	1	12	7	5
9103	22	39				6	15	1
9104	16	24				18	10	
9105	20	31		1		7	8	2
9106	45	61		2	2	9	18	
9107	25	23		1		11	4	1
9108	20	15			1	8	8	5
9109	37	39				10	8	2
9110	53	60				12	11	
	337	384	3	7	4	105	102	17

this factor the progeny are 337 normal males, 384 normal females, 10 sex intergrades, 4 triploids, 105 non-disjunctional males, 102 non-disjunctional females and 17 slender-bristled males. These results show clearly that the chromosomes of male and female Drosophila pass through phases which must be divergent in at least two particulars, chromosomal linkage and disjunction.

[1] Gowen, Marie S., and Gowen, John W., "Complete Linkage in *Drosophila Melanogaster*," 1922, *Am. Naturalist*, **56**, 286–288.

NORMAL AND "RODLESS" RETINAE OF THE HOUSE MOUSE WITH RESPECT TO THE ELECTROMOTIVE FORCE GENERATED THROUGH STIMULATION BY LIGHT[1]

BY CLYDE E. KEELER, EVELYN SUTCLIFFE AND E. L. CHAFFEE

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I. *The Character of the Rodless Retina.*—For purposes of this paper, a very simple description of the "rodless" retina will suffice. The "rodless" retina is an arrested developmental condition of the retina in which the visual elements (rods), are completely lacking and the external nuclei are highly deficient in numbers. Differentiation of fibres from the few scattered cells representing the external nuclear layer have not as yet been detected in the external molecular layer. (Failure to demonstrate these minute fibres does not prove that they may not exist.) No visual purple is secreted in this type of eye. All other parts of the retina are normal both grossly and histologically. Complete descriptions of the character have been published previously (Keeler, 1924, 1926, 1927*c*, 1927*d*) with diagrams (Keeler, 1924) and photomicrographs (Keeler, 1927*c*).

II. *The Problem.*—Since the rods, proved to be the secretory organ of visual purple (Keeler, 1927*d*), are absent in the rodless retina, one would predict either abnormal visual function or none at all.

The second possibility, complete elimination of function, has been highly favored by training tests (Keeler, 1924, 1926, 1927*c*), by stereotropic behavior (Crozier and Pincus, 1926), by orientation upon an inclined plane when negative phototropism is opposed to negative geotropism (Keeler, 1928), by attempted luminary stimulation of animals highly sensitized by injections of strychnine (Keeler, 1928, unpublished), and by flashlight photography (Keeler, 1928, unpublished).

This latter view was attacked in three papers by A. E. Hopkins (Hopkins, 1927*a*, 1927*b*, 1927*c*). This author based his argument upon training tests conducted in Münich by himself and Prof. Karl von Frisch. He contended that (Hopkins, 1927*a*, p. 343) "the animal (with rodless

retina) was apparently able to see practically as well as the others" (Hopkins, 1927*b*, p. 360) "Five mice having retinæ of this sort were successfully trained to choose white opposite red." . . . . . "It is concluded that these mice were not blind," and (Hopkins, 1927*c*, p. 490) "Five (rodless) animals were carried through series of tests which proved that they could see." These results will be discussed in the future by one of us. (C. E. K.)

Histological studies (Keeler, 1927*d*) have shown that the "rodless" retina aside from absence of rods and deficiency of external nuclei, is perfectly normal in structure and that there is no reason to believe that if a nervous impulse could be initiated in the internal nuclei or ganglionic cells, it would not have an open path to the brain.

As a crucial test for possible functional capacity of the Rodless Eye, it was decided to compare the potentials generated in the rodless and normal retinæ.

III. *Literature.*—It has been shown by Chaffee, Bovie and Hampson, 1923, Chaffee and Hampson, 1924, Meservey and Chaffee, 1927, and others that the freshly excised eye of a frog, horned toad, alligator, etc., connected to a sensitive galvanometer by electrodes from the optic stalk and cornea or exposed retina, gives a characteristic response to flashes or longer periods of exposure to light. The general form of these responses is similar in the several classes of vertebrates employed. For photographs and descriptions of the normal responses, see above papers.

Chaffee and Sutcliffe (unpublished) found that the sclerotic coat of the excised eye took on the potential of the optic nerve and hence the direct contact of one electrode with the optic nerve is unnecessary. Hartline, 1925, showed that the whole body of the animal employed took on the potential of the optic nerve, and that one led from any part of the body and the other from the cornea resulted in responses identical with those obtained from freshly excised eyes.

This latter method is highly preferable for warm-blooded animals as it leaves the eye in situ eliminating the necessity of serum medium or thermal regulator. The possibility of operational injury, usually great in small eyes, is avoided. The eye remains in its original condition indefinitely as the animal is alive and the blood supply to the retina is not impaired. In the experiments described in this paper we adopted this method.

IV. *Apparatus.*—The apparatus employed is the same as described by Meservey and Chaffee, 1927. It consists essentially of a brass eye chamber containing calomel electrodes. Cotton threads moistened in Ringer's solution serve as connections with the animal and dip into the same fluid in U tubes, connecting with the calomel electrodes. The circuit is made with a delicate Einthoven string galvanometer through a two stage resistance-coupled amplifier.

The eye chamber is enclosed in a black wooden box in the wall of which a camera shutter is installed. Light from a 500-watt lamp, passing through ground glass and a 1-inch diaphragm, falls on the shutter. The light passes through the shutter, when open, to strike upon a prism on the lid of the brass eye chamber, by which it is deflected directly down upon the eye to be tested. The total distance from light to eye is approximately 40 cm. The intensity of light striking the eye was 77.5 foot candles.

Light from a second source, passing through a projection apparatus, causes the shadow of the galvanometer string to fall upon a visible scale, at the same time that it exposes a strip of bromide paper on a revolving drum. The shadow of the galvanometer string is focused on this paper by a cylindrical lens.

A superimposed potential of 0.4 millivolt impressed upon the grid of the first tube of the amplifier for an instant at the beginning of the record, before the eye is illuminated, gives a measure with which to compare responses upon that particular record.

A second galvanometer string, the image of which appears at the top of the record, deflects simultaneously with the illumination and darkening of the eye. Deflection upward denotes "eye illuminated;" a deflection downward, "eye dark." A clock mechanism marks off time intervals of 5 seconds upon the record.

For a more detailed description of the apparatus see Chaffee, Bovie and Hampson, 1923.

V. *Procedure and Results.*—A special bakelite platform was designed to fit within the eye chamber.

The mouse to be tested was laid upon one side and bound to a piece of cardboard of convenient size and shape with strips of adhesive tape. No anæsthetic or narcotic of any kind was administered to any of the specimens. Greatest care must be exercised in order to make the mouse perfectly immobile and still not interfere with its respiration.

The skin of the face about the upturned eye was pulled back with the fingers and held in that position by a strip of adhesive tape perforated with a hole through which protruded the eye without touching the tape.

The piece of cardboard bearing the prepared mouse was placed in the eye chamber in such a position that a beam deflected by the prism would cover the whole side of the mouse's head, including the eye.

One moist thread electrode was draped over a paraffin post and its tip was placed upon the cornea of the eye. The other electrode was stuffed well into the mouth of the animal.

The lid of the eye chamber was put on and the dark box shut. The eye was allowed to become dark adapted for a definite period of time. Then the room was darkened, the shutter of the dark box opened and at a given signal the 500-watt light for illuminating the eye was switched

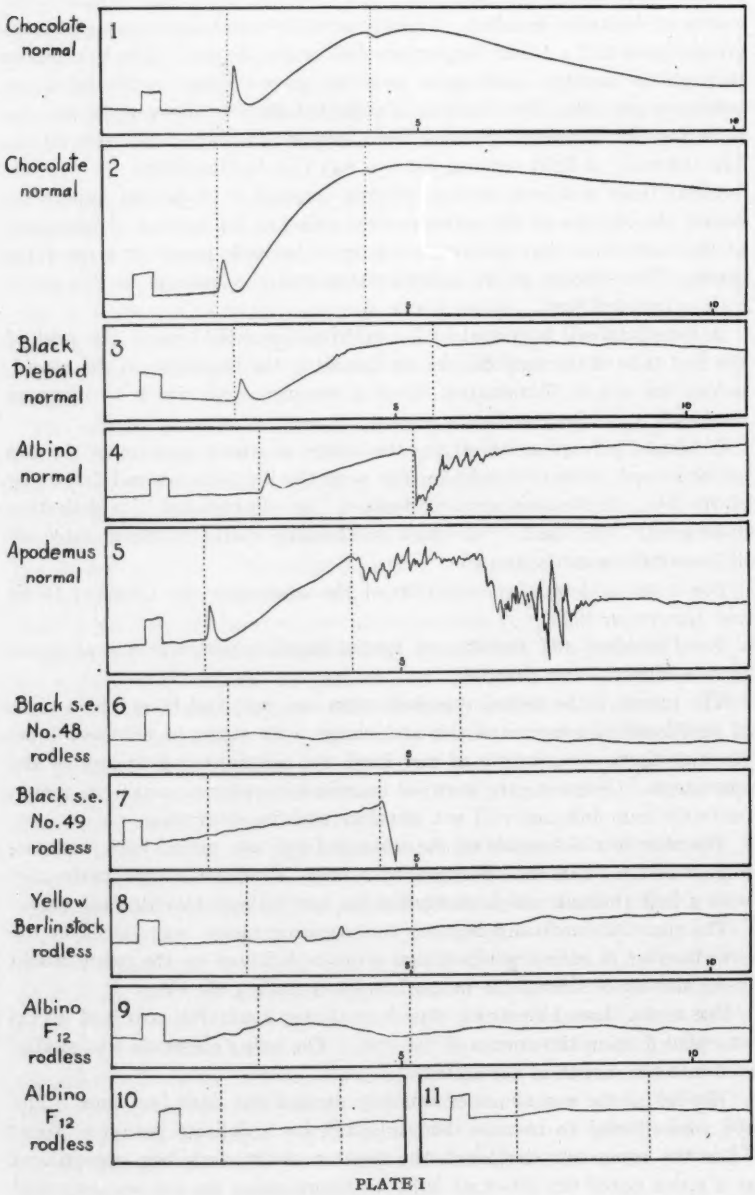
Chocolate normal
1
Chocolate normal
2
Black Piebald normal
3
Albino normal
4
Apodemus normal
5
Black s.e. No. 48 rodless
6
Black s.e. No. 49 rodless
7
Yellow Berlinstock rodless
8
Albino F_{12} rodless
9
Albino F_{12} rodless
10
11

PLATE I

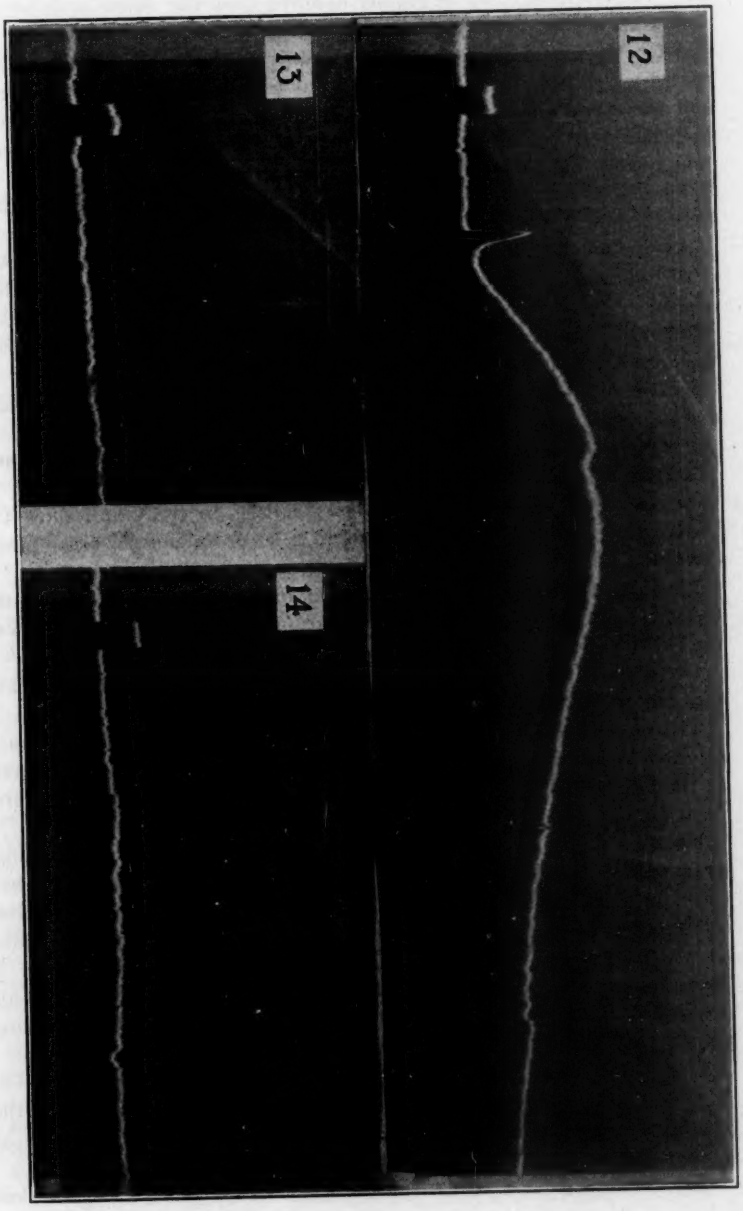

PLATE II

on. The response could be readily followed upon the visible scale by the observers.

After the response had been observed several times, a photograph record was taken. When the record was completed the box was opened and the animal observed to make certain that the electrodes were still in place after the struggling which some of the specimens made from time to time in attempting to free themselves.

The eyes tested were removed and fixed in Modified Held's fixative, imbedded in paraffin and sectioned at a thickness of 8μ.

Six rodless house mice (albino and pigmented), seven normal house mice (albino and pigmented), and one normal Apodemus sylvaticus (European wood mouse) (pigmented) were tested in irregular order. A photographic record was made for each of the seven normal house mice, the normal Apodemus, and five of the rodless mice.

Tracings of the records of four normal house mice, the Apodemus, and five rodless house mice are shown in figures 1 to 11 inclusive.

Photographs from which were made the tracings of figure 1, 10 and 11 constitute figures 12, 13 and 14, respectively.

In several of the records slight irregularities in the graph show the breathing of the animal and perhaps to a lesser extent the heart beats. See figure 7.

Irregularities due to struggling are seen in figures 4, 5 and 7, while the slight deviations of figures 3, 8 and 9 are probably due to contraction of the eye muscles.

Between the dotted lines upon each graph is represented the period of exposure to light.

The instantaneous rise and fall of potential at the beginning of each record represents the calibrating potential of 0.4 millivolt. The numerals at the baseline of all records, save figures 10 and 11, represent measurement of time in seconds.

Upon examination of the records, we find the same characteristic response to illumination in all the controls (figures 1, 2, 3, 4 and 5). In each when the light is turned on, there is a sharp rise and a more sloping decline followed by a larger and slower rise. The fact that the initial rise is distinctly unimodal is expected since we are dealing with a single type of photo receptor (rods). When the light is turned off, there is a small jog downward in the graph followed by a slight rise and a gradual lowering toward the former dark adapted level.

In contrast to the above, in all the graphs of rodless animals, figures 6, 7, 8, 9, 10 and 11 there is seen absolutely no effect of the light and the graph in each case is approximately a straight line, such variations as exist being due to extraneous causes.

The obvious general upward slope of the graph in figure 7, the less

noticeable upward slope in figures 8 and 9, and the general downward slope in figure 6 are due to drifts in the potentials of the batteries and hence are of no consequence. Similar drifts are found among the control records, especially figure 5.

Special attention should be called to figures 6 and 7, since these two graphs are from animals whose training records will be mentioned in a later paper by one of us. (C. E. K.)

Figure 8 is of interest since it represents the graph of a rodless yellow mouse from the same Berlin stock (Keeler, 1927*e*) employed by Hopkins and von Frisch. It shows absolutely no electrical response.

In no case was there any discrepancy between the photographic record and the histological details.

By breeding tests it has been shown by one of us (C. E. K. unpublished) that the Berlin Rodless and American Rodless characters are identical.

VI. *Conclusion.*—From these experiments we are forced to conclude that mice bearing "rodless" retinæ give no electrical response to stimulation by light. If one considers the electrical response a necessary concomitant of vision, then the rodless eyes are blind.

VII. *Summary.*—The potentials generated in the eyes of both pigmented and albino mice bearing normal retinæ were recorded by a very delicate and precise method. They were found to be strikingly alike and similar to the normal curves of potential in frogs, horned toads, rabbits, humans, etc.

All attempts to demonstrate and measure potential in both pigmented and albino "rodless" mice were negative.

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[1] The authors are indebted to the Bussey Institution of Harvard University for long care and maintenance of the animals employed in these experiments, and to the Milton Fund for apparatus and support of the work.

RADIATION AND RELATIVITY.[1] *I*

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1. It seems that the corpuscular theory represents well certain features of light: some properties of an elementary process of radiation are analogous to some properties of a material particle. There are differences, of course. There is the difference in velocity. In classical physics this is only a quantitative difference and therefore it is not sufficient to account for the essential dissimilarities between matter and radiation, but in relativity "velocity of light" has a preferred position and the difference in velocity alone leads to essential distinctions.[2] We propose to study systematically from the relativity point of view a particle moving with the velocity of light, following the methods by which a material particle is studied. In the present note the point of view of *special relativity* is adopted.

2. The field of a material particle can be obtained if to the general field equations we adjoin the condition that the field must possess the symmetry properties of a geometrical point. In the electrostatic case, for instance, the *general field* may be characterized by the statement that it is derived from a harmonic potential, the *symmetry property* means that this potential depends only on the distance from the particle; these two requirements together lead uniquely to the inverse square law.

3. If we want to apply this idea to radiation we have to begin by

setting up the symmetry properties of a photon. In this case the discussion will be more complicated because we cannot eliminate time: a photon cannot be transformed to rest. We have to use space-time interpretation. In space-time a particle is represented by a straight line and a vector—momentum vector—which has the direction of the line and is freely movable along it. This applies equally to a material particle and a photon. In the latter case, which interests us here, the momentum vector is of zero square; by a proper choice of coördinates we can make it the 1,0,0,1 vector and we can make the line pass through the origin. What is the field accompanying the photon? Guided by the analogy of a material particle we shall look for a field which possesses the same symmetry as the configuration representing the photon, i.e., a field which is transformed into itself by all Lorentz transformations which do not affect a vector 1,0,0,1 and a straight line of that direction through the origin.

The transformations of the group

$$\begin{aligned} x' &= x + ay + bz + {}^1/_2(a^2 + b^2)(t - x) + c \\ y' &= y + a(t - x) \qquad\qquad z' = z + b(t - x) \qquad (1) \\ t' &= t + ay + bz + {}^1/_2(a^2 + b^2)(t - x) + c \end{aligned}$$

possess the required property; but these are not all the transformations that do not affect our configuration; they also do not affect the $x = t$, $y = 0$ plane.[3] We could easily remove this restriction by considering a four-parameter group containing (1) and rotations in the y-z plane. There are two circumstances, however, which seem to indicate that (1) gives the correct expression of the symmetry of a photon. In the first place, as it will be shown later (sect. 6), there is no electromagnetic field which is invariant under this wider, four-parameter group. Secondly, we have to remember that our configuration—line and momentum vector attached to it—does not represent adequately all the features of the elementary act of radiation: polarization, i.e., the fact that space does not possess the same properties in all directions around a light particle was left out of the picture when we adopted the corpuscular point of view; the photon has to be considered as an infinitesimal plus sign rather than an infinitesimal dot; it would not be invariant under the four-parameter group. Both these circumstances lead us to regard the group (1) as adequately representing the symmetry properties of a photon or, more precisely, to call a field symmetric with respect to a photon if it is invariant under the group (1).

4. Before we try to find such fields we want to make a remark concerning the geometry of the group. Subtracting the first equation from the last we obtain $t' - x' = t - x$; this quantity, which we shall denote by u, is then an invariant of the group; two points with different values

of u cannot be transformed one into the other. But if two points have the same *u and this u is different from zero*, i.e., if two points belong to the same hyperplane $t - x = \text{const.} \neq 0$, there exists a transformation (1) which brings one into the other because in this case the equations (1) in which we regard $x, y, z, t, x', y', z', t'$ as given quantities can be solved for a, b, c. The situation is entirely different in the hyperplane $x = t$, or $u = 0$; the transformation equations become here

$$x' = x + ay + bz + c, \qquad y' = y, \qquad z' = z$$

and the anisotropy which we connect with polarization is evident.

5. According to our plan we proceed to determine an electro-magnetic field that admits the group (1), i.e., is transformed by the transformations of the group into itself. Assume that such a field exists and we know the values of its components X, Y, Z, L, M, N at a point P (for which $u \neq 0$); given any other point Q with the same u there exists (sect. 4) a transformation of the group which brings P into Q; the same transformation must carry the electromagnetic components at P into the electromagnetic components at Q; but we know the components at P; we can therefore find the components at Q if we know the law of transformation of the electromagnetic components; we assume that these components are transformed as the determinants of the matrix

$$\begin{vmatrix} x_1 & y_1 & z_1 & t_1 \\ x_2 & y_2 & z_2 & t_2 \end{vmatrix}.$$

We can say then that if the field is given at one point of a hyperplane $t-x = \text{const.}$ it is determined at all points of the same hyperplane the field will be determined in the whole space-time (with the exception of the points of the hyperplane $u = 0$), if we give the components at all points of the t-axis (with the exception of the point $x = y = z = t = 0$), and at these points the components can be given arbitrarily. The transformation which brings the point $0,0,0,u$ into a point x,y,z,t (for which $t-x = u$) is easily seen to have for its parameters

$$a = y/u, \qquad b = z/u. \qquad c = x - (y^2 + z^2)/2u.$$

Applying a transformation with these parameters to the values of X, Y, Z, L, M, N at the point $0,0,0,u$ which we denote by p, q, r, l, m, n we find

$$X = p + (q - n).\frac{y}{u} + (r + m).\frac{z}{u}$$

$$Y = q - (q - n)\frac{y^2-z^2}{2u^2} - (r + m)\frac{yz}{u^2} - p\frac{y}{u} - l\frac{z}{u}$$

$$Z = r + (r + m)\frac{y^2-z^2}{2u^2} - (q - n)\frac{yz}{u^2} + l\frac{y}{u} - p\frac{z}{u}$$

$$L = l + (r + m)\frac{y}{u} - (q - n)\frac{z}{u}$$

$$M = m - (r + m)\frac{y^2 - z^2}{2u^2} + (q - n)\frac{yz}{u^2} - l\frac{y}{u} + p\frac{z}{u}$$

$$N = n - (q - n)\frac{y^2 - z^2}{2u^2} - (r + m)\frac{yz}{u^2} - p\frac{y}{u} - l\frac{z}{u}.$$

These formulas in which p, q, r, l, m, n are arbitrary functions of u give a general electromagnetic field that admits the group (1).

6. We stop here for a moment to see whether there exists a field which in addition to being invariant under the group (1) is invariant under y-z rotations. For an electric field of that property Y and Z would be proportional to y and z, respectively, i.e., all the coefficients must vanish except p; in the same way we see that if the magnetic field is invariant under these rotations all coefficients must vanish except l. An electromagnetic field possessing this symmetry must therefore vanish identically, and this corroborates the statement made in sect. 3.

7. Our next step is to subject this field to Maxwell's equations (for empty space). The result of the simple calculations (we need to consider the field only at the points of the t-axis) is that pu^2, lu^2, $(q-n)u$, $(r+m)u$ must be constants. Denoting these constants by $\alpha, \beta, \gamma, \delta$, respectively, and introducing the notations

$$\tfrac{1}{2}(q + n) = f(u), \qquad \tfrac{1}{2}(r - m) = g(u)$$

we obtain a general Maxwell field which allows the group (1). We present it as a sum of two fields; one, involving the arbitrary functions $f(u)$ and $g(u)$,

$$X = 0,\ Y = f(u),\ Z = g(u),\ L = 0,\ M = -g(u),\ N = f(u)$$

is the ordinary plane wave. The second is

$$X = \frac{1}{u^2}(\alpha + \gamma y + \delta z)$$

$$Y = \frac{\gamma}{2u} - \frac{1}{u^3}\left[\gamma\frac{y^2 - z^2}{2} + \delta yz + \alpha y + \beta z\right]$$

$$Z = \frac{\delta}{2u} + \frac{1}{u^3}\left[\delta\frac{y^2 - z^2}{2} - \gamma yz + \beta y - \alpha z\right]$$

$$L = \frac{1}{u^2}(\beta + \delta y - \gamma z)$$

$$M = \frac{\delta}{2u} - \frac{1}{u^3}\left[\delta\frac{y^2 - z^2}{2} - \gamma yz + \beta y - \alpha z\right]$$

$$N = -\frac{\gamma}{2u} - \frac{1}{u^3}\left[\gamma\frac{y^2 - z^2}{2} + \delta yz + \alpha y + \beta z\right].$$

This field seems to deserve a close study. For the present, however, we limit ourselves to the following remarks. If we separate again time and space we may say that the field moves together with the photon remaining unchanged. There is a plane singularity accompanying the photon—the plane which contains the photon and is perpendicular to the direction of its motion is a singular plane of the field; we might think of it as playing the rôle of a wave front. There is, however, no periodicity in the field. Further, the field is not "self-conjugate:" the electric and magnetic vectors are not at right angles and of equal length; but this condition is approached asymptotically as we approach the singular plane because then the terms not in the square brackets may be neglected.

[1] Presented before the Chicago meeting of the American Mathematical Society, April 15, 1927.

[2] Compare *Physical Review*, **31**, March, 1928, pp. 448–49.

[3] These PROCEEDINGS, **13**, February, 1927, pp. 29–31.

ON THE ABSOLUTE MAGNITUDES OF THE CLASS M STARS

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Communicated May 15, 1928

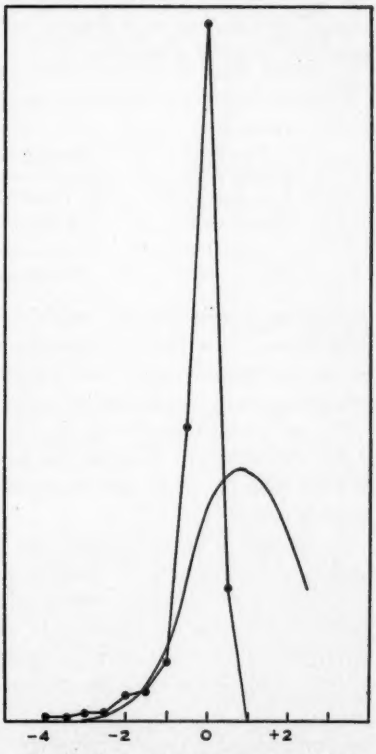
In *Harvard Circular* 273 the writer has published an investigation of the absolute magnitudes of the brighter class *M* stars. While this article was in press a paper by van Rhijn[1] appeared on the same subject. Later Adams, Joy and Humason[2] have published a list of absolute magnitudes of 410 class *M* giants and dwarfs, while the system of absolute magnitudes underlying their paper has been discussed by Strömberg.[3] On account of the wide discrepancy between the results of these three investigations it may be of interest to compare them.

Adams, Joy and Humason have used essentially all the *M* stars in Boss' catalogue north of declination $-30°$, a number of fainter stars in the selected areas and eight stars in the double cluster in Perseus. In the present paper we shall not consider these cluster stars. The mean absolute magnitude and the dispersion for the first two groups are, respectively:

271 Boss stars	$M = -0.24 \pm 0.04$	$\sigma = 0.64 \pm 0.03$
34 fainter stars	$M = -0.25 \pm 0.09$	$\sigma = 0.55 \pm 0.07.$

We may say therefore that, spectroscopically speaking, the Boss stars and the fainter *M* stars form a homogeneous group, and the results derived by Adams, Joy and Humason may thus be taken as representative of the *M* giants in space.

Van Rhijn's investigation deals strictly with the luminosity and density distribution of the *M* stars in space, and is based upon the Boss stars and a large number of fainter stars for which proper motions have been determined at various observatories, but for which practically no radial velocities are available. The writer has shown that at least some of the catalogues giving proper motions of fainter *M* stars differ systematically from Boss.[4] Van Rhijn's material cannot be considered homogeneous, and this lack of homogeneity is fatal since the absolute magnitudes of the faint *M* stars are thus made dependent upon their own proper motions and the radial velocities of the brighter stars.



A comparison of frequency curves of absolute magnitudes of giant *M* stars. The smooth curve, which stops at +2.5, is van Rhijn's curve for all *M* stars in space. The line with the much higher maximum represents the spectroscopic absolute magnitudes determined at Mount Wilson. This latter curve includes a dispersion, due to errors of observation, of 0.64.

In figure 1 are compared the luminosity curve derived by van Rhijn, and the curve resulting from the data given by Adams, Joy and Humason. No attempt has been made to equalize the total number of stars represented by the two curves, which would be difficult to do since van Rhijn's curve is not tabulated beyond +2.5, but obviously does not stop there. It is evident, however, that no change in the total number of stars can remove the discordance. When van Rhijn's material is examined, it is at once obvious that any discrepancy would be in the direction observed, since his employment of too large proper motions for the fainter stars would naturally result in too large a dispersion in absolute magnitude. On the other hand, the small dispersion in the Mount Wilson magnitudes is surprising. A little closer inspection of their results may throw some light upon this.

Adams, Joy and Humason have called attention to the fact that the

supergiants of class M, those stars whose absolute magnitude is brighter than -1.0, show a pronounced galactic concentration. For this reason the 271 Boss stars in the Mount Wilson list have been arranged in four groups according to their galactic latitude, and the mean absolute magnitude and dispersion determined for each group. These data are given in table 1.

TABLE 1

LIMITS OF β	$\bar{M}$	σ	n
0 to ± 15	-0.71 ± 0.14	1.08 ± 0.10	55
± 15 to ± 30	-0.17 ± 0.06	0.50 ± 0.04	63
± 30 to ± 50	-0.14 ± 0.04	0.34 ± 0.03	82
± 50 to ± 90	-0.05 ± 0.04	0.31 ± 0.03	71
0 to ± 20	-0.59 ± 0.11	0.97 ± 0.08	78
± 20 to ± 90	-0.10 ± 0.03	0.34 ± 0.02	193

Since there is only one star brighter than -1.0 with a galactic latitude of more than 17°, a second arranging was made, merely dividing the stars into a low latitude group and a high latitude group. The data for these two groups are given in the last two lines of table 1.

We notice that according to the Mount Wilson spectroscopic magnitudes, it is sufficient to know that a star of class M has a galactic latitude of more than 20° to be able to predict its absolute magnitude with a mean error of only 0.34.

In *Mount Wilson Contributions*, No. 327, Strömberg discusses the Mount Wilson system of absolute magnitudes, and derives a mean error of 0.94 for a spectroscopic determination of absolute magnitude of a supergiant M star from one single plate, and 0.67 for that of an ordinary giant. Strömberg himself declares that parallaxes derived for the ordinary M giants on the assumption that the absolute magnitude is constant would be of "nearly" the same accuracy as would be attained by determining them from the examination of the individual spectrum. The analysis of the data made in the present note shows that the assumption that the absolute magnitude of an M giant is constant, and equal to -0.6, will give as good a parallax as a single spectroscopic determination when the galactic latitude of the star is less than twenty degrees. For stars of higher latitude where supergiants are statistically absent, the estimate on the assumption that the absolute magnitude is constant and equal to -0.1, will have four times the weight of a spectroscopic estimate made from one single plate.

The aggregate mean error of the absolute magnitudes of *all* M giants observed at Mount Wilson is 0.64 ± 0.03, smaller than the smallest error of spectroscopic measurement—which is, according to Strömberg, 0.67 for an ordinary giant. Nevertheless, it seems that the line-intensity criteria as used by the Mount Wilson observers do distinguish between giants and supergiants, although they cannot be considered as closely

correlated with the small differences in absolute magnitude among the ordinary giants.

The dispersion in absolute magnitude among the class M giants cannot now be determined, but will have to await both the accumulation of reliable proper motions for the apparently fainter stars, free from systematic errors such as are now present, and the determination of radial velocities for the fainter stars in numbers sufficiently large to permit statistical utilization.

[1] *Publ. Kapteyn Astr. Lab. Groningen*, No. 38, **1925.**
[2] *Contrib. Mt. Wilson Obs.*, No. 319, **1926.**
[3] *Contrib. Mt. Wilson Obs.*, No. 327, **1927.**
[4] *Harv. Ann.*, **81**, No. 2, 1923.

ON THE CONSTANCY OF THE LIGHT OF RED STARS, WITH FORTY NEW VARIABLES OF THIS CLASS

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It is well known that many of the red stars are variable in light, the conspicuous example being α Orionis or Betelgeuse which changes irregularly to the extent of fifty per cent or more in the course of a few months, but no period or law of the variation of this star has been found in studies extending over many years. There are numerous other such objects in the sky, but the red stars are the most difficult to observe visually. Their very color sets them apart from other objects, and the presence of haze or moonlight, combined with the Purkinje phenomenon, gives rise to subjective effects which make it unusually difficult to establish real changes unless they are quite large, say thirty or forty per cent.

The observations described in the present note have been made in the years 1926–1928 as a first survey or test of several scores of red stars for small variations in light. The instrument was the photo-electric photometer attached to the 15-inch refractor of the Washburn Observatory. The observing list was intended to include all stars of spectrum M in the Harvard classification which are north of $-10°$ declination and brighter than visual magnitude 6.0, corresponding to a limit of about the seventh photo-electric magnitude. The red or M-stars were to be observed in groups of three or four, including one or more yellow K-stars for comparison. If only two stars were compared with each other there would be no way of telling which is the variable, while because of the necessary check on the atmospheric extinction a group of four stars is about as large

as can be conveniently observed. At first the list contained some groups of four M-stars, or of three M's and one K, but when such groups were observed and no two objects were found which gave a constant difference of magnitude, it was necessary to break up the groups and use additional K-stars as standards of reference.

For each observation of say three stars, a, b and c, the measures were taken in the symmetrical order $a\ b\ c\ a\ c\ b\ a$, and a similar grouping was made for four stars. Measures of the same group were taken ordinarily about once a month, and the minimum number of observations was three, the average number up to date being about five, extending over at least two months. At the present writing 164 M-stars have been tested using 165 K-stars as standards. For the M-stars the detailed classification from Mount Wilson has been used.[1] On the basis of the intensity of the titanium oxide bands the sequence is $M0$, $M1$, ...$M6$, which is also the scale of increasing redness, while the divisions beyond $M6$ are reserved for the long-period variable stars which have enormous light changes, up to several thousand-fold, with significant complications in their spectra. As K-stars we have included $K0$, $K2$ and $K5$ of the Harvard classes, with a few $G5$.

TABLE 1

RANGE OF VARIATION OF RED STARS

	G5-K5	*M0*	*M1*	*M2*	*M3*	*M4*	*M5*	*M6*	TOTAL *M*-STARS
0^m.000–0^m.069	165	29	26	22	16	1	0	0	94
0.070–0.099	...	3	1	8	7	2	0	1	22
0.100–	...	2	2	9	8	11	7	2	41
Visual variables	...	0	0	2	1	1	2	1	7
Totals	165	34	29	41	32	15	9	4	164

In deciding upon what is to be called a variable star, there arises the difficulty that there is no sharp dividing line between small variations of the stars and the unavoidable errors of the photometric measures. From long experience it is known that a probable error of one observation of something like one per cent is to be expected. Therefore, if on comparison of observations on two dates one star is found to be discordant by ten or twenty per cent while the measures of the other objects in the group are in close agreement, then either there is some mistake in the work or the star in question is a variable. An additional observation will usually confirm the variation unless indeed one of the comparison stars changes, when still further measures may have to be made. After some consideration we have adopted 0.100 magnitude as the minimum range to be called a variable, 0.070 to 0.099 magnitude is labeled as suspicious, while for variations of 0.069 magnitude or less the amount due to the star is likely to be so small that it is not worthwhile to follow it further.

The results up to May 1, 1928, are given in table 1. Under each spectrum class is given the number of stars within the corresponding limits of variation. The known visual variables have all a range of half a magnitude or more.

The first thing that comes out of the table is that of the 165 *K*-stars not a single one has been even suspected of variation, for in every group the *K*-stars have always been constant when referred to each other. The limit of 0.070 for suspicious objects was adopted before it was known that all of the *K*-stars would have a smaller range.

TABLE 2

RED STARS WITH VARIATION GREATER THAN 0.20 MAGNITUDE

BOSS NO.	NAME	VISUAL MAGNITUDE	SPECTRUM	ABSOLUTE MAGNITUDE	RANGE	NO. OBS.	REMARKS
81	47 Piscium	5.3	*M*3	−0.6	0.25	6	
660	45 Arietis	5.9	*M*6	0.0	0.41	8	
698	ρ Persei	3.7	*M*4	−0.7	....	..	Visual
1335	119 Tauri	4.7	*M*2	−3.0	0.25	6	
1468	α Orionis	0.9	*M*2	−4.3	....	..	Visual
1561	η Geminorum	3.8	*M*3	−0.4	....	..	Visual
1606	ψ^1 Aurigae	5.1	*M*0	−2.7	0.41	..	Many measures
1856	51 Geminorum	5.3	*M*4	−0.4	0.45	7	
1871	25 H. Camel.	5.1	*M*4	−0.1	0.26	9	
2915	56 Leonis	6.0	*M*5	−0.6	0.28	3	
3584	H. R. 5219	5.0	*M*2	−0.3	0.21	7	
3630	H. R. 5299	5.4	*M*4	−0.7	0.22	10	
3827	H. R. 5589	4.9	*M*5	0.0	0.22	7	
4125	10 Herculis	6.0	*M*4	−0.3	0.21	6	
4201	ξ Herculis	5.0	*M*6	0.0	....	..	Visual
4373	α Herculis	3.5	*M*5	−1.6	....	..	Visual
4800	δ^2 Lyrae	4.5	*M*4	−1.4	0.25	14	
4814	R Lyrae	4.4	*M*5	−0.6	....	..	Visual
5125	13 Sagittae	5.6	*M*4	−0.4	0.24	4	
5234	H. R. 7804	6.0	*M*5	0.0	0.24	5	
5593	μ Cephei	4.4	*M*2	−3.0	....	..	Visual
5650	H. R. 8383	5.4	*M*2*e*	−2.0	0.22	8	
5940	β Pegasi	2.6	*M*2	−0.4	0.34	10	Suspected visual
5986	χ Aquarii	5.1	*M*5	0.0	0.28	10	

Among the *M*-stars, the tendency to variability with increasing redness is quite distinct. Of the 63 *M*0- and *M*1-stars, 55 are constant, 4 suspicious and only 4 variable; while bunching *M*4, *M*5 and *M*6 together, we find in a total of 28 stars that only 1 is constant, 3 are suspicious and the remaining 24 are variable. Classes *M*2 and *M*3 are intermediate between these extremes. In regard to the one constant and three suspicious stars of *M*4 and *M*6, it is probable that further observations will show that these are variables, so we may say that any distinctly red star, i.e., of classes *M*4 to *M*6, is almost certainly a variable to the extent of ten per cent or more.

In the title of this paper the number of new variables is given as forty, but from the comparison with the yellow stars it is evident that most of

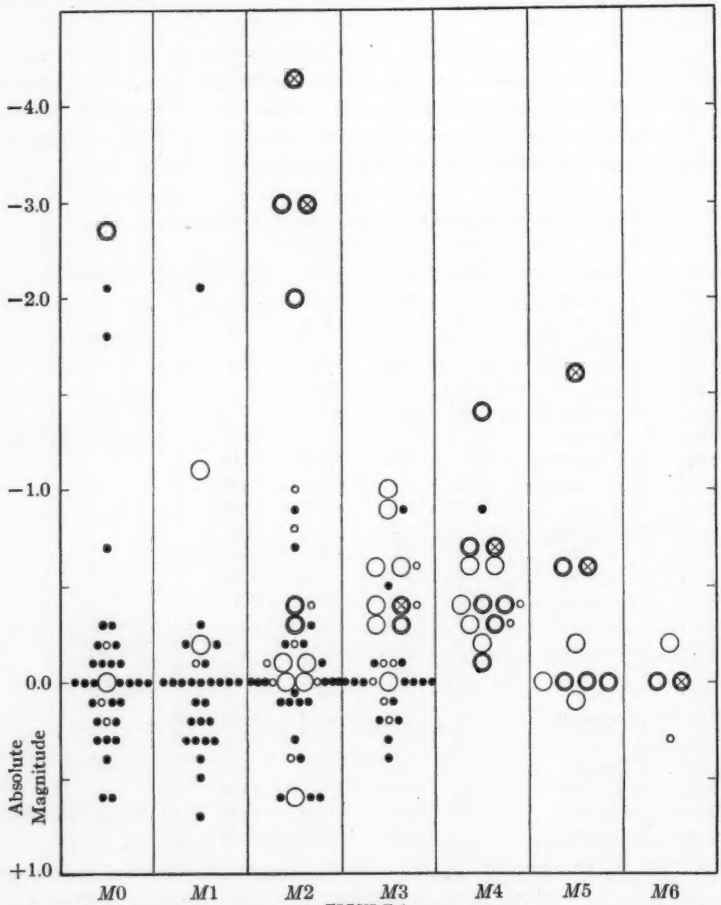

FIGURE 1

Photometric tests of Red Stars

Dots.....Range $0^m.000$ to $0^m.069$
Small Circles.....Range $0^m.070$ to $0^m.099$
Large Circles.....Range $0^m.100$ to $0^m.199$
Double Circles.....Range $0^m.200$ and more
Crossed Circles.....Visual variables

the suspicious objects are probably variable, so that the number of new ones is nearer sixty. Since all of these objects are visible to the naked

eye, the names of the known visual variables and the new variables with large range may be of interest to some workers in this field, and these stars are given in table 2. The visual magnitudes are taken from the Harvard catalog and usually refer to a sort of mean brightness. The spectrum and absolute magnitude are from Mount Wilson. The range and number of photo-electric observations are given for the new variables, while most of the visual variables have also been tested and confirmed here.

The results of tables 1 and 2 are also shown in figure 1, which is the upper right-hand corner of Russell's well-known diagram of absolute magnitude against spectral type. As the sun's absolute magnitude is about +5, it is seen that all of these objects are giant stars, the faintest object in our list being forty times as bright as the sun.

In a discussion of this work with Dr. Adams at Mount Wilson last summer, he suggested that we would probably find an increasing tendency to variability as we passed from fainter to brighter absolute magnitude, and this has turned out to be the case where there are enough stars from which to judge. In classes *M*2 and *M*3 the variables clearly average brighter than the constant stars. In all the classes, stars brighter than absolute magnitude −1.0 are variable except the three stars of spectra *M*1 and *M*2, of which there are only a few measures. Either extreme luminosity or extreme redness seems to go with variability.

From the irregular behavior of the previously known variable red stars, it is not expected that even approximate periods can be determined for these new objects. We have scrutinized the measures of the forty stars to make a first guess at the probable length of a cycle from maximum to maximum, or from minimum to minimum. Although no exactness can be claimed, it seems clear that for about one-half of the stars any period must be greater than 100 days, for two or three stars it is less than 50 days, and the remainder are doubtful.

The significance of the result of this photometric study of red stars in connection with current theories of evolution is obvious. According to all theories the trend of a diffuse giant star would be toward contraction and increased surface temperature, which means a motion from right to left on the diagram in figure 1, with little change up or down in absolute magnitude. The very red stars may be called the young stars, and in their early stages the output of radiation seems to be quite irregular, and changes of ten or twenty per cent and more within a few months are not uncommon. In the course of years or even of centuries, however, the variation probably keeps within rather definite limits. No such star has been known to drop out of sight, and certainly no newcomer has burst forth to challenge Betelgeuse in the winter sky. As the red stars continue to contract and grow hotter during cosmic time, they pass over

to the fairly steady M states and then on to the yellow class K, where in early adult life the worst irregularities of youth are left behind.

[1] Adams, Joy and Humason, *Mount Wilson Contributions*, No. 319, **1926.**

FAMILY TRAITS AS DETERMINED BY HEREDITY AND ENVIRONMENT

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An investigation of immigrants and their descendants undertaken by me in 1910, showed certain differences between those born abroad and those born here.[1] The first study was based on a comparison of gross series of individuals born abroad and of others born here. It seemed possible that these differences were due to a different composition of the two series; that for some reason those born abroad originated from localities or from social strata not identical with those to which the American series belonged. For this reason the further investigation was based on a comparison of parents and their own children, and of fraternities, some of whose members were born abroad while others were born here. As a result of these studies, I stated that under new environment, stature, head form and transversal diameter of the face undergo changes. These results have frequently been challenged; but the mere claim that the traits studied *must* be stable cannot disprove the data collected. Since these are not based on theory but on observations, the only way to refute them would be to prove either an error of method or to disprove the results by a new series of observations. Neither the one nor the other has been done. On the contrary, a check on a small scale, made by Dr. Guthe in Boston, showed the same results. Later on I have been able to show similar changes of type in Porto Rico, although in this case parents and their own children could not be compared. The essential proof, therefore, is lacking. Dr. Hrdlička's investigation of old Americans, that is to say, of those whose ancestors have lived in the United States for three generations or more, shows also changes in facial form. However, in this case also, the proof is not absolutely conclusive because the ancestral series is not available. In 1926 Miss Ruth O. Sawtell and Dr. Isabel G. Carter repeated the examination of Sicilians in New York and Philadelphia, particularly for the reason that in 1912, when the earlier investigation was concluded, no adult American-born Sicilians were available. It is still difficult to obtain an adequate number of these, but

the results of the two new series agree with those obtained previously. The tests were so arranged that an assistant recorded the statistical information in regard to the place of birth and immigration, etc., while the observer did not know whether he was dealing with American-born or foreign-born.

Recently the problem of the stability of form has been taken up experimentally by Eugen Fischer who has been able to prove that the headform of animals changes with the kind of feeding.

The fundamental question at issue is in how far those measurements that are considered as fundamental racial traits may be considered as stable and in how far they are influenced by environmental conditions. We have ample evidence that the bulk of the body, stature and weight are strongly influenced by external conditions. It is not so evident that skeletal forms determined in early life will be so influenced. I have shown in 1914[2] that the low correlative value for stature, as compared to the much higher correlative value for head index between parents and children, may be interpreted as due to disturbances of hereditary similarity. When outer conditions, like varying state of health and other accidental causes, influence each individual in a fraternity independently from the other, the similarity and the coefficient of correlation will decrease. In order to illustrate this I assumed at that time, arbitrarily, that the head index was entirely uninfluenced by environmental conditions and that the hereditary transmission of head index and stature followed the same rules. Under these arbitrary suppositions it can be shown what part of the variability of stature is due to heredity and what part to environment. This was meant only as an illustration of the method of approach. An objection must be raised against this method. (See farther on, page 499.)

If a particular feature of the body is inherited in any type of fixed Mendelian ratio it follows necessarily that the distribution of forms of the offspring of a pair of parents and, with it, their average depend upon the constitution of the parents. If the parents in one group are less heterozygous than those in another, the series of offspring will, on the whole, be more uniform. In the F_2 generation, and in the simplest form of a recessive and a dominant unit character, the offspring of the recessive form will have no more variability than the parental form, while the offspring of the dominant form will vary considerably. It follows from this that, if a particular measure is grouped in such a way that in one class families are assembled in which parents have exceedingly low values of a measure, in another class those in which both parents have exceedingly high values, and in still another those in which both parents have a medium value for the same measure, the variability of the children must be different for each group. I have investigated from this point of view the head indices with the result that for Sicilians and Central Italians the variability of

cephalic index of fraternities remains unaltered, no matter whether the two parents have an index of low, medium or high value. This indicates that we may say that the head index must be considered as determined by a large number of factors acting in different directions.

In a previous publication I have considered the variability of the cephalic index of a large number of fraternities in its relation to the differences between the parents.[3] It appeared in this case that the variability was the greater, the greater the difference between the parents. This would indicate a tendency for the cephalic index to split up in Mendelian fashion. The question, however, is not definitely settled. More material is necessary to give a final result.

It is necessary to emphasize here that a group of individuals of a bodily form, selected from a given population, is functionally not identical with a selected group of identical form selected from another population.

I may illustrate this by the example of children of Sicilians and Bohemians whose parents have a head index varying from 79.5–82.5. The children of the former have an average index of 79.3, those of the latter 83.0; while the averages for the parents are, respectively, 80.6 and 81.0, showing a slightly different grouping of individuals in the parental groups. This is merely an expression of the different genetic composition of these series, but it must be borne in mind, for it is often erroneously assumed that the same form wherever found has the same significance. It is not possible to analyze a stable population as though it were an agglomeration of types that may be identified with other similar neighboring types. It is a unit, and each individual must be conceived as a variant of the local type.

Two populations may be alike in the general distribution of forms and still the composition may be fundamentally distinct. In one we may have a large number of similar family lines and of strongly variable fraternities; in the other strongly varying family lines and uniform fraternities. It is necessary to separate these two types of variability, which may easily be done by determining the variability of the fraternities reduced to fraternities of an infinite number of individuals. I found for Sicilians, for instance, for the head index a variability of family lines of ± 2.3; while the variability of the fraternities amounted to ± 2.4. Among the Chippewa Indians, an old inbred community, the variability of the family lines is ± 1.77, that of the fraternities ± 3.32.[4] It has been found throughout, as might be expected, that in inbred stable populations the family lines are very much alike, while the variability of the fraternities depends upon their more or less homogeneous ancestry. In unstable populations brought together recently from different parts of a large district, the family lines vary very much, while fraternities may be quite uniform.

A consideration of the relation between fraternal variability and the

variability of family lines shows also that the coefficient of fraternal correlation depends solely upon the variability of family lines and the general composition of the population. When the family lines are alike, the coefficient of fraternal correlation is low, when they are diverse, it is high.

We assume all families to consist of an infinite number of members of a fraternity. The family means will vary around the average of the total population by amounts to be designated by x; the individual members of each fraternity will deviate from their family mean by amounts to be designated by y. The variability of family means may be designated by σ_l, that of the fraternities by σ_{fr}. We make the simplifying assumption that the variabilities of all the family lines are the same. The total variability of the population is

$$\sigma^2 = \sigma_l^2 + \sigma_{fr}^2.$$

Then

$$r_{fr} = \frac{[(x_1 + y')(x_1 + y'')]}{\sigma^2}.$$

Since the number of members of the fraternity is taken as infinite the products yy are negligible and we find

$$r_{fr} = \sigma_l^2/\sigma^2. \qquad (1)$$

It must be borne in mind that the family lines are merely averages of the fraternities with an infinite number of siblings. Environmental factors that are purely accidental will have no influence upon their family means. It is however not necessarily implied that these family means are hereditary, nor that if the averages of the descendants should agree with the averages of the parental family line, that the members of the fraternity as parents should all represent the same parental family line.

If all the family lines were equal the fraternal correlations would be zero. The coefficient has no biological significance whatever. The coefficient of paternal correlation is also affected by the composition of the population but in a more complex manner. [See (5).]

The argument referred to before (page 497), can be used only when the family lines for stature and head index, or of any other two measurements that are compared, have the same ratios of σ_l and σ.

We have seen before that observations of headform and several other traits indicate that each type is subject, to a greater or less extent, to environmental influences. This problem may also be attacked in another manner. I shall illustrate this by the example of Johannsen's beans. He found that in a self-fertilized, pure line the average weight of beans of parents and descendants was identical. Beans of the same weight, but belonging to different lines, were compared with the average weight

of their descendants and he found a regression to the general phenotype. This is due to the greater frequency of genotypes near the average of the phenotype. The coefficient of parental regression is a function of the distribution of genotypes and of their non-hereditary variability. In Johannsen's series[5] the mean square variability of nineteen pure lines of mother beans (weight in centigrams), is $\sigma_h = \pm\sqrt{39}$, the total variability $\pm\sqrt{152}$. There remains therefore for the non-hereditary mean square variability the value of $\sigma_n = \pm\sqrt{113}$. When we select therefore beans of the same weight, but belonging to different lines, they will represent an average line different from the selected one because many combinations of hereditary and non-hereditary deviations give the same value and all of these are combined in the series. A simple consideration shows that, if both variations are subject to chance only, the average value of these hereditary lines will be

$$[z] = x\,\sigma_h^2/\sigma^2 \qquad (2)$$

and its mean square variability

$$\sigma_h^2\sigma_n^2/\sigma^2 + x^2\sigma_n^4/\sigma^4. \qquad (3)$$

Here the hereditary element may contain also an environmental factor to which the whole group is exposed. Introducing here the values found before for Johannsen's line, namely, $\sigma_h^2 = 39$ and $\sigma^2 = 152$, we find that the average hereditary line for a deviation x will be $0.26\,x$. Since, according to Johannsen, the average weight for the lines of descendants of a pure line is equal to the weight of the mother beans, this must also be the average value for the descendants and the coefficient of regression would be 0.26. On page 135 of his book, Johannsen finds this value as 0.257.

The study of the coefficient of correlation for parents and children shows that they cannot be harmonized on the assumption that the parental type determines entirely by heredity the type of the descendant. The theory of correlation, which holds good no matter whether we are dealing with Mendelian or other inheritance, demands that the average variability of the offspring of two parents shall have a value $\sqrt{1-2r^2}$, provided there is no selective mating. When we compare the actual values for fraternal variability, that is to say, the mean square variation of members of a fraternity around the family mean, this is found to be very much smaller than the theory demands. For the head indices of Sicilians, for instance, the mean square variability is $\pm\sqrt{5.6}$ while the theory demands $\pm\sqrt{9.3}$. This can be explained only by the assumption that the selected deviation of the parents with which the form of the descendants is correlated belongs to a great variety of family lines. We may consider this in the same way as we considered the phenomena observed by Johannsen.

The father and mother who have the selected deviations x from the

average of the population represent many different lines, the average of which are according to equation (2),

$$[z] = x\,\sigma_h^2/\sigma^2.$$

When the father has the deviation x, the mother y, and both affect the offspring in the same way, the average line will be

$$[z] = \frac{\sigma_h^2}{\sigma^2} \cdot \frac{(x + y)}{2}. \qquad (4)$$

The available data do not show that the average of the fraternities descended from these parents differs from the average of the family lines to which father and mother, respectively, belong. The multiplicity of determining factors and of lines is so great that no one type appears as dominant. The variability of the fraternities descended from these parents may also be assumed as constant. These assumptions are approximations, for undoubtedly individually the fraternities will have different variabilities. It does not seem, however, that their averages change considerably with changing values of x and y. If we knew the hereditary value of the family lines and the variability of the descendant fraternities the correlation between parents and children could be determined.

In a stable population the children of a couple with the hereditary deviations x and y cannot all become the parents of a family line intermediate between x and y, because, if this were the case, the mean square variability of the family lines would be reduced in each successive generation. Mendelian splitting of family lines makes it possible that the distribution of both family lines and of fraternities remains stable. In other words: a population can be stable, only, if there are certain types of reversion to ancestral forms.

Accepting the simplified assumptions stated before, the average of the fraternities will be

$$[v] = \frac{\sigma_h^2}{\sigma^2} \cdot \frac{(x + y)}{2}.$$

Since in a stable population the values of σ_x, σ_y and σ_v must be on the average the same, the coefficient of parental correlation will be

$$r_p = \frac{\sigma_h^2}{2\sigma^2} \qquad (5)$$

and may be assumed as equal for father and mother. In case of selective mating $\frac{r_p}{1 + r_{f,m}}$ must be substituted, when $r_{f,m}$ is the coefficient of correlation between father and mother.

Only in exceptional cases can the average of the hereditary family lines be equal to the averages of the family lines as determined from the coefficient of correlation of fraternities. [See (1).]

Since we found that

$$r_{fr} = \sigma_l^2/\sigma^2$$

it would follow, if σ_l and σ_h were equal, that the fraternal correlation must be double the parental correlation. As will be seen from the tables at the end of this note, this is not the case. Since accidental environmental conditions disappear in the family lines as determined from infinitely large fraternities, the differences between the two must be due to the differences of new family lines established by the members of a fraternity. If we call this value σ_i

$$\sigma_i^2 = \sigma_l^2 - \sigma_h^2.$$

Or substituting from equations (1) and (5)

$$\sigma_i^2 = \sigma^2(2r_p - r_{fr}).$$

The total hereditary variability is under the conditions mentioned before $2r_p\sigma^2$; the non-hereditary variability must be the remainder of the total variability

$$\sigma_n^2 = \sigma^2 - \sigma_l^2.$$

It follows that whenever the distribution of family lines is the same as that of individuals the non-hereditary variability is zero: in other words, that the form is entirely determined by heredity. This is the case in the simple forms of Mendelian heredity in stable populations with non-selective mating. However, in this case the variabilities of fraternities are not constant.

As examples I give the values for the head index of Sicilians and of North American Negroes and Mulattoes, the latter kindly furnished to me by Dr. Melville J. Herskovits, also the values for the bizygomatic width of face of Bohemians and for stature for Galton's series which was calculated by Karl Pearson in 1895.[6] In all of these I have used the average values for all variabilities and coefficients, disregarding the slight differences of those resulting from the comparison of fathers and mothers with sons and daughters, and of brothers and sisters in their mutual relations.

	HEAD INDEX							WIDTH OF FACE		
	BOHEMIANS	POLES	HEBREWS	SICILIANS 1910	SICILIANS 1926	NEGROES	CENT. ITAL.	BOHEMIANS	SICILIANS	STATURE ENGLISH
σ^2	10.90[1]	11.81[1]	9.04[1]	10.36[1]	10.90[1]	10.75[1]	12.47[1]	33.8[2]	25.5[2]	6.58[3]
r_{fr}	0.355	0.435	0.285	0.412	0.315	0.31	0.495	0.61	0.47	0.403
r_p	0.155	0.237	0.240	0.232	0.195	0.23	0.29	0.33	0.32	0.336

[1] Units. [2] Sq. mm. [3] Sq. inches.

From these we obtain:

	HEAD INDEX							WIDTH OF FACE		
	BOHEMIANS	POLES	HEBREWS	SICILIANS 1910	SICILIANS 1926	NEGROES	CENT. ITAL.	BOHEMIANS	SICILIANS	STATURE ENGLISH
σ^2_{fam}	3.87[1]	5.14[1]	2.58[1]	4.26[1]	3.43[1]	3.30[1]	6.17[1]	20.62[2]	11.98[2]	2.65[3]
σ^2_{ind}	0.00	0.41	1.76	0.54	0.82	1.64	1.06	1.69	4.34	1.77
$\sigma^2_{non\text{-}her}$	7.03	6.26	4.70	5.56	6.65	5.81	7.29	11.49	9.24	2.16

[1] Units. [2] Sq. mm. [3] Sq. inches.

These examples are intended to indicate a method by which the non-hereditary elements may be distinguished. The method contains many simplifying assumptions that do not correspond to reality. It seems however possible that the ultimate problem may be attacked by a further elaboration of this method.

[1] *Changes in Bodily Form of Descendants of Immigrants*, New York, 1912.

[2] *Zeitschrift Ethnologie*, **45**, pp. 624, et seq.

[3] *Changes in Bodily Form*, etc., p. 154.

[4] *American Anthropologist*, **18** (1916), pp. 1–9.

[5] *Elemente der exakten Erblichkeitslehre*, W. Johannsen, 1909, pp. 138, 139.

[6] "Mathematical Contributions to the Theory of Evolution—III," *Royal Society* 1896, **99**, 270, 281.